

Initial Jump and Recovering Jump in the S&P 500 Index Returns: A Jump-Recovering-Switching Approach

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This paper develops a three-state jump-recovering-switching model (JRS model), coupling jump processes and a regime-switching methodology, to investigate the dynamic patterns and statistical properties of initial and recovering jumps in S&P 500 stock index returns from 2002-2015. The empirical findings show that a “directional effect” and a “magnitude effect,” two jump phenomena in the returns process from the perspective of overreaction, are empirically supported.

Keywords: jump, regime switching, stock returns

JEL classification: C22, G10, G11

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1 Introduction

Financial markets sometimes generate significant extreme changes, or so-called jumps in asset prices (Das, 1998, 2002; Chernov *et al.*, 2003; Eraker *et al.*, 2003; Eraker, 2004; and Maheu and McCurdy, 2004). A number of empirical and theoretical studies support the existence of jumps and their substantial impact on financial decisions, such as asset pricing, risk management, portfolio allocation, and hedging (Merton, 1976; Naik and Lee, 1990; Bates, 1996a, 1996b, 2000; Bakshi *et al.*, 1997, 2000; Duffie *et al.*, 2000; Das, 2002; Liu *et al.*, 2003; Johannes, 2004; Lee, 2009). The occurrence of jumps is mainly motivated by discontinuous information vis-à-vis macroeconomic news, earnings announcements, or rare events (Clark, 1973; Ane and Geman, 2000; Maheu and McCurdy, 2004; Rangel, 2011.) Further, Maheu and McCurdy (2004) suggest that the number of jumps in stock returns series depends on unusual news in the market (responding to important events), leading to extreme movements in returns. The presence of jumps implies that market participants may react to unanticipated news systematically over time. Moreover, from a market microstructure perspective, Evans and Lyons (2002) argue that the occurrence of jumps results from portfolio shifts caused by private, non-common knowledge, news sources such as direct interdealer transactions, and customer orders. Dealers observe interdealer order flows to learn about the shift. As the market gradually aggregates the information, transactions between dealers and the non-dealer public may create jumps in prices of assets being traded. Though modeling jump patterns in asset prices is not an easy task, the Poisson distribution provides a simple approach which has proven to be useful in empirical studies (Jorion, 1988; Vlaar and Palm, 1993; Das, 1998; Chan and Maheu, 2002; Chernov *et al.*, 2003; Maheu and McCurdy, 2004).

Jump processes can be employed to capture the large movements in stock markets that continuous models are unable to accommodate. But the interplay between the various jump patterns over time is not trivial, and standard jump specifications are unable to replicate these patterns. Indeed, jumps in asset prices in one day seem to exist in a short period and increase the probability of observing successive recovering jumps, and then the returns go back to a normal level. This implies that a positive jump could come after a negative jump immediately, and vice

versa. Bondt and Thaler (1985), Jegadeesh (1990), Lo and MacKinlay (1990), and Jegadeesh and Titman (1993) show that stock returns exhibit reversals in longer-run intervals. In this paper, we define the jump occurring at the beginning as the “initial jump” and the following as the “recovering jump”. Though the occurrence of the initial jump could have a substantial and immediate impact on returns, the following recovering jump could fully or partially offset the effect of the initial jump. Taking this pattern into account in the model not only sheds more lights on the relationship between the initial jump and the recovering jump but is also useful for correctly estimating the impacts of jumps over a shorter period.

Another interesting question arising from these dynamics is why the recovering jump occurs. If the initial jump is caused by unusual incoming news as previous literature suggests, we posit that investors could overreact to that unusual news so that the actual initial jump is larger than what it should be based on the scale and scope of the unusual news. When investors realize that they have overreacted to the unusual news, they will try to make a considerable correction to mitigate their previous decision. As a result, the recovering jump occurs and offsets (wholly or partially) the initial jump. On the other hand, if the initial jump occurs without any incoming news or information, as is sometimes the case, Chan (2003) argues that returns could still exhibit reversal in the subsequent period because investors sometimes overact to initial extreme price movements per se (not to the news) and later make corrections voluntarily.

Accordingly, we summarize two jump phenomena in the returns process from the perspective of overreaction, as shown in Bondt and Thaler (1985) and Brown and Harlow (1988). First, large price changes (initial jump) will be followed by price reversals (recovering jump) in the opposite direction, namely the “directional effect”. Second, the larger the initial jump, the greater the subsequent recovering jump, denoted as the “magnitude effect.” Bondt and Thaler (1985) and Brown and Harlow (1988) have argued that the stock market overreacts to information such as past earnings and stock prices. Using cross-sectional approaches, both studies found that price reversals will occur in the longer-run for undervalued stocks that have experienced large price changes over the preceding five years. Compared to Bondt and Thaler (1985) and Brown and Harlow (1988), our paper offers two new contributions. First, focusing on short-run overreaction, we consider that price

changes are mainly due to short-term information arrivals. Therefore, our model is configured by discrete jumps to account for the arrival of unusual information with continuous normal innovations capturing the arrival of regular information. Thus, we use market factors such as stock returns, trading volumes, and volatilities to explain the probabilities of staying in different regimes of price changes. Second, we use a time-series approach and model the dynamics of stock returns series directly rather than observing the price reversals of stocks cross-sectionally as in Bondt and Thaler (1985) and Brown and Harlow (1988). Thus, our model can directly estimate the expected magnitudes of jumps and the expected durations of jumps. To that end, this paper develops a model to study the properties of initial and recovering jumps in the S&P 500 index. As a result, based on our model, we can investigate directional and magnitude effects from the perspective of overreaction. Moreover, we can use this model to detect how investors overreact to unusual news or to extreme price changes per se.

2 Methodology

The purpose of this paper is to develop a jump-recovering-switching model (JRS model) for the returns process. The JRS model allows discrete jumps with time-varying jump intensities as in Chan and Maheu (2002) and the parameters in the model are state-dependent with a latent state variable s_t governed by a 3-state first-order Markov process. The model of stock returns R_t is specified as:

$$R_t = \mu_{s_t} + \varepsilon_{s_t,t} + J_{s_t,t}, \quad (1)$$

where μ_{s_t} is the mean in regime s_t ; $\varepsilon_{s_t,t}$ is the residual $\varepsilon_{s_t,t} | I_{t-1} \sim N(0, \sigma_{s_t}^2)$, capturing the process of normal news arrivals and assumed to be contemporaneously independent of $J_{s_t,t}$; I_{t-1} represents the information set up to $t-1$; $\sigma_{s_t}^2$ is the variance; $J_{s_t,t}$ is the discrete jump component,¹ capturing the process of unusual news arrivals in regime s_t and defined as follows:

¹By definition, $J_{s_t,t}$ are jump components and $\varepsilon_{s_t,t} = R_t - \mu_{s_t,t} - J_{s_t,t}$ so $\varepsilon_{s_t,t}$ contains no jumps.

$$J_{s_t,t} = \sum_{i=0}^{n_{s_t,t}} Y_{s_t,t,i}, \quad (2)$$

where $Y_{s_t,t,i}$ is the jump size and $Y_{s_t,t,i} | I_{t-1} \sim N(\theta_{s_t}, \delta_{s_t}^2)$; $n_{s_t,t}$ denotes the discrete counting process governing the number of jumps that occur between $t-1$ and t and follows a Poisson distribution:

$$P(n_{s_t,t} = j_{s_t} | s_t = s, I_{t-1}) = \frac{\exp(-\lambda_{s_t}) \lambda_{s_t}^{j_{s_t}}}{j_{s_t}!}, j_{s_t} = 0, 1, 2, \dots, \quad (3)$$

with jump intensity λ_{s_t} .

To incorporate the properties of the initial jump and the recovering jump into the 3-state JRS model, we make two assumptions regarding the switching process between regimes. First, in the no-jump regime ($s_t = 1$), jumps are not allowed. Therefore, the jump component $J_{1,t}$ is assumed to be zero. Second, in our specification, stock returns begin in the no-jump regime ($s_t = 1$) and can remain in that regime with transition probability p_{11} or move to the initial-jump regime ($s_t = 2$) with $p_{12} = 1 - p_{11}$, where the transition probability is defined as follows:

$$p_{ij} = P(s_t = j | s_{t-1} = i). \quad (4)$$

The no-jump regime cannot move directly to the recovering-jump regime ($s_t = 3$); thus, $p_{13} = 0$. Once in the initial-jump regime, it is not possible to revert to the no-jump regime ($p_{21} = 0$). Instead, it either remains in the initial-jump regime (i.e., jump again in the same direction as in the initial-jump regime) with p_{22} or moves to the recovering-jump regime ($s_t = 3$) with $p_{23} = 1 - p_{22}$. Consequently, when in the recovering-jump regime, it could either stay in the recovering-jump regime (i.e., jump again in the same direction as in the recovering-jump regime) with p_{33} or revert back to the no-jump regime with $p_{31} = 1 - p_{33}$. To summarize, our transition probability matrix is specified as

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} p_{11} & 1-p_{11} & 0 \\ 0 & p_{22} & 1-p_{22} \\ 1-p_{33} & 0 & p_{33} \end{pmatrix}. \quad (5)$$

To apply maximum likelihood estimation, we follow Lee (2009) and employ the filter in Hamilton (1989, 1994) for regime switching and the filter in Chan and

Maheu (2002) for jumps. Considering the conditional density of returns of being in regime s and information set I_{t-1}

$$f(R_t|s_t = s, I_{t-1}) = \sum_{j_{s_t}=0}^{\infty} f(R_t, n_{s_t,t} = j_{s_t}|s_t = s, I_{t-1}), \quad (6)$$

where

$$f(R_t, n_{s_t,t} = j_{s_t}|s_t = s, I_{t-1}) = f(R_t|n_{s_t,t} = j_{s_t}, s_t = s, I_{t-1}) P(n_{s_t,t} = j_{s_t}|s_t = s, I_{t-1}). \quad (7)$$

$f(R_t|n_{s_t,t} = j_{s_t}, s_t = s, I_{t-1})$ in Equation (7) denotes the conditional density of returns in regime s given that j_{s_t} jumps occur and the information set I_{t-1} is normally distributed, and can be defined as

$$f(R_t|n_{s_t,t} = j_{s_t}, s_t = s, I_{t-1}) = \frac{1}{\sqrt{2\pi(\sigma_{s_t}^2 + j_{s_t}\delta_{s_t}^2)}} \exp\left[-\frac{(R_t - \mu_{s_t,t} - j_{s_t}\theta_{s_t})^2}{2(\sigma_{s_t}^2 + j_{s_t}\delta_{s_t}^2)}\right]. \quad (8)$$

When returns data at time t have been observed, we can infer the *ex post* probability that j_{s_t} jumps occur in regime s at time t with the filter proposed by Chan and Maheu (2002), and thus the *ex post* probability can be defined as follows:

$$P(n_{s_t,t} = j_{s_t}|s_t = s, I_t) = \frac{f(R_t|n_{s_t,t} = j_{s_t}, s_t = s, I_{t-1})P(n_{s_t,t} = j_{s_t}|s_t = s, I_{t-1})}{f(R_t|s_t = s, I_{t-1})}, \quad (9)$$

where $P(n_{s_t,t} = j_{s_t}|s_t = s, I_{t-1})$ is defined in Equation (3), $f(R_t|n_{s_t,t} = j_{s_t}, s_t = s, I_{t-1})$ is defined in Equation (8), and $f(R_t|s_t = s, I_{t-1})$ is defined in Equation (6).

In addition, the state probability $P(s_t = s|I_{t-1})$ in Equation (7) is defined as

$$P(s_t = s|I_{t-1}) = \sum_{i=1}^3 p_{is}P(s_{t-1} = i|I_{t-1}), \quad (10)$$

where p_{ij} is defined in Equation (4). To ensure that p_{ij} lies between 0 and 1, we impose a restriction on p_{ij} with a logistic function. That is, we specify $p_{ij} = 1/[1 + \exp(-p_{ij}^*)]$ and estimate the parameter p_{ij}^* in the procedure of maximum likelihood estimation. In the empirical results, we also report the value of \hat{p}_{ij} by calculating $1/[1 + \exp(-\hat{p}_{ij}^*)]$.

According to Hamilton's filter for the switching process, when returns data at time t have been observed, we can infer the *ex post* probability of regime s . We first define the information set up to $t-1$ and t respectively as $I_{t-1} = [R_{t-1}, R_{t-2}, \dots, R_0]$ and $I_t = [R_t, I_{t-1}]$. By Bayes' Theorem, we have $(s_t = s | R_t, I_{t-1}) = f(R_t, s_t = s | I_{t-1}) / f(R_t | I_{t-1})$, where $f(R_t, s_t = s | I_{t-1}) = f(R_t | s_t = s, I_{t-1}) f(s_t = s | I_{t-1})$ by Bayes' Theorem again. Finally, we get the *ex post* probability of regime s given the information set up to t as follows:

$$P(s_t = s | I_t) = \frac{f(R_t | s_t = s, I_{t-1}) P(s_t = s | I_{t-1})}{f(R_t | I_{t-1})}, \quad (11)$$

where $f(R_t | s_t = s, I_{t-1})$ and $P(s_t = s | I_{t-1})$ are defined as per Equation (6) and Equation (10), respectively. Equation (9) and Equation (11) are important components of time-varying jump dynamic and state processes in our JRS model.

Finally, the conditional density of returns given information set I_{t-1} can be derived as follows:

$$f(R_t | I_{t-1}) = \sum_{s=1}^3 f(R_t | s_t = s, I_{t-1}) P(s_t = s | I_{t-1}), \quad (12)$$

and all parameters can be estimated by maximizing the log-likelihood function:

$$\log L = \sum_{t=1}^T \log [f(R_t | I_{t-1})]. \quad (13)$$

Once we obtain all the estimates in our model, since the magnitudes of initial jump and recovering jump are defined as $MAG_2 = E(J_{2,t} | s_t = 2, I_{t-1}) = \lambda_2 \theta_2$ and $MAG_3 = E(J_{3,t} | s_t = 3, I_{t-1}) = \lambda_3 \theta_3$, respectively, the recovery rate (*RR*) can be calculated by $|\hat{\lambda}_3 \hat{\theta}_3 / \hat{\lambda}_2 \hat{\theta}_2|$. Moreover, the duration of jumps in state i (DUR_i) can be calculated by $1/(1 - \hat{p}_{ii})$.

To investigate if the probability of remaining in state i , $p_{i,t} = P(s_t = s | I_{t-1})$ is correlated with the factors generated by the price change such as daytime returns (DR_t), change in trading volume (DV_t), and range volatility ($Range_t$), we specify a regression equation to be estimated by ordinary least squares (OLS) and censored regression

$$p_{i,t} = a_{i,0} + a_{i,10} DR_t + a_{i,11} DR_{t-1} + a_{i,20} DV_t + a_{i,21} DV_{t-1} + \quad (14)$$

$$a_{i,30}Range_t + a_{i,31}Range_{t-1} + a_{i,4}p_{i,t-1} + e_{i,t}.$$

3 Data and Empirical Results

This paper investigates initial and recovering jumps in the S&P 500 index. All data are obtained from Datastream. The sample period spans 2002 to 2015. S&P 500 index returns $R_t = 100 \times [\log(SP_t^c) - \log(SP_{t-1}^c)]$ where SP_t^c is the daily closing price at time t . Moreover, the factors generated by the price change such as daytime returns (DR_t), change in trading volume (DV_t) and range volatility ($Range_t$) are calculated respectively by $DR_t = 100 \times [\log(SP_t^c) - \log(SP_t^o)]$, $DV_t = 100 \times [\log(Volume_t) - \log(Volume_{t-1})]$ and $Range_t = 100 \times [\log(SP_t^h) - \log(SP_t^l)]$ where $Volume_t$ is trading volume, and SP_t^o , SP_t^h and SP_t^l are the opening, highest, and lowest prices, respectively.

Table 1 reports descriptive statistics for R_t during three sub-periods, 2002–2006, 2007–2010, and 2011–2015. The mean returns are positive during 2002–2006 and 2011–2015 and negative during 2007–2010 (the subprime crisis period). Volatility measures of R_t , proxied by standard deviations, range from 0.991 (2011–2015) to 1.706 (2007–2010). The subprime crisis increased stock market volatility.

Table 1. Descriptive Statistics

	2002-2006	2007-2010	2011-2015
Observations	1303	1045	1155
Mean	0.021	-0.007	0.038
Standard deviation	1.000	1.706	0.991
Percentile (5%)	-1.553	-2.721	-1.556
Percentile (95%)	1.602	2.385	1.487

Table 2. JRS Coefficient Estimates

	2002-2006	2007-2010	2011-2015
μ_1	0.027 (0.024) [0.251]	0.042* (0.025) [0.089]	0.091** (0.029) [0.002]
σ_1^2	0.512** (0.040) [0.000]	1.096** (0.111) [0.000]	0.314** (0.059) [0.000]
μ_2	0.023 (0.326) [0.943]	0.076 (0.392) [0.846]	0.302** (0.028) [0.000]
σ_2^2	1.231 (0.870) [0.157]	18.585** (1.506) [0.000]	2.942** (0.224) [0.000]
θ_2	-1.054** (0.266) [0.000]	-1.307* (0.775) [0.092]	-1.022** (0.039) [0.000]
δ_2^2	0.159 (1.018) [0.876]	2.615 (4.313) [0.544]	0.462** (0.044) [0.000]
λ_2	0.939** (0.220) [0.000]	0.505** (0.087) [0.000]	0.480** (0.045) [0.000]
μ_3	0.013 (0.134) [0.921]	-0.210 (0.203) [0.302]	-0.118** (0.026) [0.000]
σ_3^2	1.708** (0.149) [0.000]	3.940** (0.984) [0.000]	0.946** (0.111) [0.000]
θ_3	0.901** (0.094) [0.000]	1.844** (0.135) [0.000]	0.373** (0.032) [0.000]
δ_3^2	0.550** (0.030) [0.000]	2.255** (0.192) [0.000]	0.126** (0.062) [0.041]
λ_3	0.623** (0.117) [0.000]	0.115 (0.073) [0.118]	0.421** (0.031) [0.000]
p_{11}^*	2.863** (0.249) [0.000]	5.493** (0.362) [0.000]	3.538** (0.352) [0.000]
p_{22}^*	$p_{11}=0.946$ -0.163 (0.559) [0.770]	$p_{11}=0.996$ 2.989** (0.508) [0.000]	$p_{11}=0.972$ 2.056** (0.425) [0.000]
p_{33}^*	$p_{22}=0.459$ 0.452* (0.241) [0.061]	$p_{22}=0.952$ 4.090** (0.297) [0.000]	$p_{22}=0.887$ 3.058** (0.111) [0.000]
	$p_{33}=0.611$	$p_{33}=0.984$	$p_{33}=0.955$

Numbers in parentheses are standard errors and those in brackets are p-values.

** and * denote significance at the 5% and 10% level, respectively.

Coefficient estimates derived from the JRS model are presented in Table 2. All estimates are obtained via maximum likelihood estimation. Table 2 shows that most of the estimates are statistically significant (at either the 5% or 10% level). The jump size parameters θ_2 and θ_3 are negative and positive respectively during each sub-period, supporting the “directional effect” whereby a positive price reversal

follows a large negative price change. Moreover, the absolute values of θ_2 and θ_3 are the largest while the jump intensity parameters λ_2 and λ_3 are the lowest during the 2007-2010 subprime crisis period.

Table 3 reports durations and magnitudes for the initial and recovering jumps as well as the recovery rate during each period. The durations of the initial-jump state (DUR_2) during each period are 1.85, 20.87, and 8.81 days, respectively. The durations of the recovering-jump state (DUR_3) during each period are 2.57, 60.75, and 22.28 days, respectively. The magnitudes of the initial jump (MAG_2) are -0.99, -0.66, and -0.49 during each period, respectively. The magnitudes of the recovering jump (MAG_3) are 0.56, 0.21 and 0.16 during each period, respectively. The larger the magnitude of the initial jump, the larger the magnitude of the subsequent recovering jump, supporting the “magnitude effect.” The recovery rate is 57% during 2002-2006, 32% during 2007-2010, and 33% during 2011-2015.

Table 3. Duration, Magnitude, and Recovery Rate

	2002-2006	2007-2010	2011-2015
DUR_2	1.85	20.87	8.81
DUR_3	2.57	60.75	22.28
MAG_2	-0.99	-0.66	-0.49
MAG_3	0.56	0.21	0.16
RR	57%	32%	33%

The magnitudes of the initial jump and the recovering jump are defined as $MAG_2 = E(J_{2,t}|S_t = 2, I_{t-1}) = \lambda_2\theta_2$ and $MAG_3 = E(J_{3,t}|S_t = 3, I_{t-1}) = \lambda_3\theta_3$, respectively. The recovery rate (RR) can be calculated by $|\hat{\lambda}_3\hat{\theta}_3/\hat{\lambda}_2\hat{\theta}_2|$. Moreover, the duration of time spent in state i (DUR_i) can be calculated by $1/(1 - \hat{p}_{ii})$.

Table 4 presents coefficient estimates derived from OLS and censored regressions, configured in Equation (14), to identify the price factors affecting the probability of being in the initial- and recovering-jump states. The results are very similar across both of these regressions. We find that DR_t , $Range_t$, and $Range_{t-1}$ are the general factors affecting the probability of being in an initial-jump state ($p_{2,t}$) across the three periods. But the magnitude becomes smaller during the 2007–2010 period. The price factors exhibit a smaller impact on $p_{3,t}$ during the 2011–2015 period than during 2002–2006. Moreover, only $Range_t$ affects the probability of being in the recovering-jump state.

Table 4. OLS and Censored Regression Coefficient Estimates

	2002-2006		2007-2010		2011-2015	
	OLS	Censored	OLS	Censored	OLS	Censored
$a_{2,0}$	-0.010 ** (0.005) [0.025]	-0.010 ** (0.005) [0.025]	0.004 *** (0.001) [0.006]	0.004 *** (0.001) [0.005]	0.000 (0.004) [0.938]	0.000 (0.004) [0.938]
$a_{2,10}$	-6.703 *** (0.218) [0.000]	-6.703 *** (0.218) [0.000]	-0.117 *** (0.043) [0.007]	-0.117 *** (0.043) [0.007]	-0.927 *** (0.187) [0.000]	-0.927 *** (0.186) [0.000]
$a_{2,11}$	1.003 *** (0.268) [0.000]	1.003 *** (0.267) [0.000]	-0.017 (0.044) [0.688]	-0.017 (0.043) [0.687]	0.516 *** (0.189) [0.006]	0.516 *** (0.188) [0.006]
$a_{2,20}$	-0.026 ** (0.012) [0.035]	-0.026 ** (0.012) [0.035]	0.003 (0.004) [0.509]	0.003 (0.004) [0.507]	0.009 (0.011) [0.410]	0.009 (0.011) [0.408]
$a_{2,21}$	0.002 (0.012) [0.874]	0.002 (0.012) [0.873]	0.004 (0.004) [0.331]	0.004 (0.004) [0.329]	0.024 ** (0.010) [0.019]	0.024 ** (0.010) [0.018]
$a_{2,30}$	5.279 *** (0.380) [0.000]	5.279 *** (0.379) [0.000]	0.168 ** (0.079) [0.034]	0.168 ** (0.079) [0.033]	2.842 *** (0.364) [0.000]	2.842 *** (0.362) [0.000]
$a_{2,31}$	-1.961 *** (0.398) [0.000]	-1.961 *** (0.397) [0.000]	-0.391 *** (0.079) [0.000]	-0.391 *** (0.078) [0.000]	-2.410 *** (0.361) [0.000]	-2.410 *** (0.360) [0.000]
$a_{2,4}$	0.618 *** (0.021) [0.000]	0.618 *** (0.021) [0.000]	1.004 *** (0.004) [0.000]	1.004 *** (0.004) [0.000]	0.969 *** (0.009) [0.000]	0.969 *** (0.009) [0.000]
$a_{3,0}$	-0.047 *** (0.005) [0.000]	-0.047 *** (0.005) [0.000]	-0.003 *** (0.001) [0.005]	-0.003 *** (0.001) [0.005]	-0.005 * (0.003) [0.074]	-0.005 * (0.003) [0.073]
$a_{3,10}$	3.080 *** (0.235) [0.000]	3.080 *** (0.234) [0.000]	-0.020 (0.043) [0.640]	-0.020 (0.043) [0.637]	-0.419 *** (0.147) [0.004]	-0.414 *** (0.147) [0.005]
$a_{3,11}$	-4.011 *** (0.258) [0.000]	-4.011 *** (0.257) [0.000]	-0.058 (0.043) [0.180]	-0.058 (0.043) [0.178]	-0.780 *** (0.149) [0.000]	-0.786 *** (0.149) [0.000]
$a_{3,20}$	-0.014 (0.013) [0.269]	-0.014 (0.013) [0.267]	0.002 (0.004) [0.620]	0.002 (0.004) [0.624]	-0.002 (0.009) [0.779]	-0.002 (0.009) [0.776]
$a_{3,21}$	0.031 ** (0.013) [0.016]	0.031 ** (0.013) [0.016]	0.002 (0.004) [0.588]	0.002 (0.004) [0.592]	-0.004 (0.008) [0.641]	-0.004 (0.008) [0.609]
$a_{3,30}$	5.220 *** (0.407) [0.000]	5.220 *** (0.406) [0.000]	0.123 * (0.074) [0.099]	0.123 * (0.074) [0.096]	-0.053 (0.264) [0.841]	-0.066 (0.263) [0.802]
$a_{3,31}$	0.922 ** (0.437) [0.035]	0.922 ** (0.435) [0.034]	0.100 (0.074) [0.174]	0.101 (0.073) [0.170]	0.915 *** (0.257) [0.000]	0.924 *** (0.256) [0.000]
$a_{3,4}$	0.782 *** (0.016) [0.000]	0.782 *** (0.016) [0.000]	0.995 *** (0.002) [0.000]	0.995 *** (0.002) [0.000]	0.990 *** (0.004) [0.000]	0.990 *** (0.004) [0.000]

Numbers in parentheses are standard errors and those in brackets are p-values.

***, **, and * denote significance at the 1%, 5%, and 10% level, respectively.

Figure 1 plots the log return of S&P 500 and the smoothed probability of the recovering-jump regime with the shaded area indicating the days where the smoothed probability of the initial-jump regime is larger than 0.5. We find that the

smoothed probability of the recovering-jump regime is high immediately after initial-jump regimes. Moreover, Figure 1 also confirms that volatility is a key factor affecting the probability of initial and recovering-jump regimes.

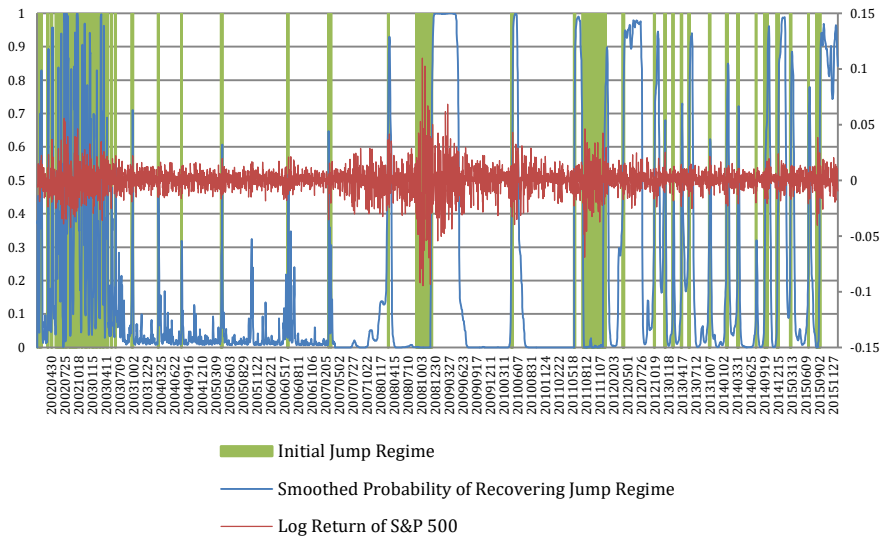


Figure 1. Initial-Jump Regime, Recovering-Jump Regime, and Log Return of S&P 500

4 Conclusions

In this study, we have proposed a three-state jump-recovering-switching model and it has been successfully tested against S&P 500 index returns spanning a 14-year period (from 2002-2015). Based on our findings, the initial jump is followed by a recovering jump in the opposite direction, which supports the “directional effect.” The data also indicate that the larger the initial jump, the greater the subsequent recovering jump, which provides further support for the “magnitude effect” discussed in the existing literature.

Further, several interesting jump phenomena are identified for the subprime crisis period. Namely, the durations of each regime in this period are substantially longer, and the magnitudes of jumps and recovery rates are smaller compared to the two other sub-periods demarcated in our sample. These findings indicate that our

model can be successfully used to trace and measure the impact and jump transition performance of a target period.

While this research remains preliminary, there is ample scope to apply similar JRS model methodologies to detect, or even forecast, jump behavior vis-à-vis duration, magnitude, and recovery rates by configuring time-varying specifications of jump size, intensity, and transitional probabilities.

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Appendix

This Appendix provides a detailed elaboration of the log-likelihood function used in this paper. The model of stock returns R_t is specified as:

$$R_t = \mu_{s_t} + \varepsilon_{s_t,t} + J_{s_t,t}, \quad (15)$$

where μ_{s_t} is the mean in regime s_t ; $\varepsilon_{s_t,t}$ is the residual and $\varepsilon_{s_t,t} | I_{t-1} \sim N(0, \sigma_{s_t}^2)$; I_{t-1} represents the information set up to $t-1$; $\sigma_{s_t}^2$ is the variance; $J_{s_t,t}$ is the discrete jump component in regime s_t and defined as follows:

$$J_{s_t,t} = \sum_{i=0}^{n_{s_t,t}} Y_{s_t,t,i}, \quad (16)$$

where $Y_{s_t,t,i}$ is the jump size and $Y_{s_t,t,i} | I_{t-1} \sim N(\theta_{s_t}, \delta_{s_t}^2)$; $n_{s_t,t}$ denotes the discrete counting process governing the number of jumps that occur between $t-1$ and t and follows a Poisson distribution as follows:

$$P(n_{s_t,t} = j_{s_t} | s_t = s, I_{t-1}) = \frac{\exp(-\lambda_{s_t}) \lambda_{s_t}^{j_{s_t}}}{j_{s_t}!}, j_{s_t} = 0, 1, 2, \dots, \quad (17)$$

with jump intensity λ_{s_t} .

Let $f(R_t|n_{s,t} = j_{s_t}, s_t = s, I_{t-1})$, the conditional density of returns in regime s , given that j_{s_t} jumps occur and are normally distributed, can be defined as follows:

$$f(R_t|n_{s,t} = j_{s_t}, s_t = s, I_{t-1}) = \frac{1}{\sqrt{2\pi(\sigma_{s_t}^2 + j_{s_t}\delta_{s_t}^2)}} \exp\left[-\frac{(R_t - \mu_{s_t,t} - j_{s_t}\theta_{s_t})^2}{2(\sigma_{s_t}^2 + j_{s_t}\delta_{s_t}^2)}\right]. \quad (18)$$

Therefore, the conditional density of R_t in regime s can be specified by integrating out the number of jumps as follows:

$$f(R_t|s_t = s, I_{t-1}) = \sum_{j_{s_t}=0}^{\infty} f(R_t, n_{s,t} = j_{s_t}|s_t = s, I_{t-1}). \quad (19)$$

Consequently, the conditional density of returns given information set I_{t-1} can be derived as follows:

$$f(R_t|I_{t-1}) = \sum_{s=1}^3 f(R_t|s_t = s, I_{t-1})P(s_t = s|I_{t-1}), \quad (20)$$

where

$$P(s_t = s|I_{t-1}) = \sum_{i=1}^3 p_{is}P(s_{t-1} = i|I_{t-1}); \quad (21)$$

p_{ij} is the transition probability, defined as follows:

$$p_{ij} = P(s_t = j|s_{t-1} = i). \quad (22)$$

Based on Bayes' Theorem, the *ex post* probability of regime s can be defined as follows:

$$P(s_t = s|I_t) = \frac{f(R_t|s_t = s, I_{t-1})P(s_t = s|I_{t-1})}{f(R_t|I_{t-1})}. \quad (23)$$

Consequently, by merging Equations (17)-(23), the log-likelihood function with parameters $\{\mu_{s_t}, \sigma_{s_t}^2, \theta_{s_t}, \delta_{s_t}^2, \lambda_{s_t}, p_{ij}\}$ can be formulated by the following:

$$\log L = \sum_{t=1}^T \log[f(R_t|I_{t-1})]. \quad (24)$$

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