Journal of Economics and Management, 2018, Vol. 14, No. 1, 33-50

Indirect Utility Reflecting Anxiety and Flexibility of Choice

Somdeb Lahiri[*]

School of Petroleum Management, PD Petroleum University, India

This paper introduces the concept of an indirect utility function reflecting anxiety of choice and an indirect utility function reflecting flexibility of choice. Using a total of six different properties, we provide separate axiomatic characterisations of these two kinds of indirect utility functions.

Keywords: indirect utility function, anxiety of choice, flexibility of choice

JEL classification: D11, D12

1 Introduction

Beginning with the work of Koopmans (1964) and subsequently with that of Kreps (1979), a very large amount of literature has investigated two-stage (or dynamic) choice problems. Such problems in decision making can be represented through the following method (called the standard approach): In the first stage, the decision maker chooses a menu of options/alternatives/elements, while in the second stage, the decision maker chooses the best alternative from the menu chosen in the first stage. Thus, for instance (as in Kreps (1990)), the choice at the first stage may be among several restaurants, and the second stage choice may be viewed as selecting an item from the menu card of the restaurant chosen in the first stage. We refer to the menu as an opportunity set in this study and related works.

There are two distinct ways in which this related literature has grown. One approach, which follows Kreps (1988, 1990) (Proposition 13.3 and Proposition 4.1,

[*]Correspondence to: School of Petroleum Management, PD Petroleum University, Raisan, Gandhinagar - 382 007, Gujarat, India; E-mail: somdeb.lahiri@gmail.com.

respectively), retains the original interpretation of the problem, with later contributions mainly axiomatisations of what can realistically be referred to as an "indirect utility function" (see Arrow (1994), Malishevsky (1997), Lahiri (2003)) or its generalisations as in Ryan (2014) and Quin (2015). The second approach relates to the dynamic choice problem pioneered by Sen (1988), which concerns itself with the freedom of choice at the first stage of a two-stage process. Since then, a significant strand of the literature has pursued this problem as in Sen (1990, 1991), Pattanaik and Xu (1990, 1997, 1998), Puppe (1995, 1996), Klemisch-Ahlert (1993), and Gekker (2001), to mention a few. This second approach argues that "more is better", and hence there is greater freedom of choice when there is more to choose from. Thus, greater flexibility is desirable in this mode of thinking.

There is a common misconception that the indirect utility approach and the flexibility approach are about one and the same thing, so that the freedom of choice literature augments the work begun by Kreps (1979). While mathematically the two structures appear similar, their lines of investigation are completely different. This is best understood when we contrast freedom of choice or flexibility with an alternative possibility, namely "anxiety of choice" or "choice overload". The genesis of this idea can be traced to a paper by psychiatrist Zbigniew Lipowski (Lipowski (1970)), in which this viewpoint is summarised as follows:

"*I maintain that it is specifically the overabundance of attractive alternatives, aided and abetted by an affluent and increasing complex society, that leads to a conflict, frustration, unrelieved appetitive tension, more approach tendencies and more conflict- a veritable vicious cycle".*

This cycle very likely has "*far reaching and probably harmful effects on the mental and physical health of affected individuals*". The conclusion that Lipowski arrives at is that an overabundance of good scenarios is the main source of anxiety that he observed around him, whereby here in a land of abundance "*the fate of Buridan's ass haunts us*". (Our source is Konnikova (2014).) The psychologist Barry Schwartz carries forward this line of argument in his book (Schwartz (2004)). Baucells and Sarin (2012) discuss this phenomenon by citing a study that observes the behaviour of customers in two different choice situations. Customers were given the opportunity to taste samples of various kinds of jam that were more or less similar in taste, and then they were offered a one-dollar discount if they decided to make a purchase. “When six varieties of jam were made available, 30 percent of the customers actually bought a jar, whereas when twenty-four varieties were available, more people came to the table but only 3 percent of them bought a jar.”

Both views on the availability of alternatives to make a choice from are thus plausible, but under different circumstances. Greater flexibility of choice or “more is better” seems appropriate in the context of deprivation and less developed economies, whereas “anxiety of choice” or “choice overload” seems suitable in the context of plenty and abundance. Bavetta *et al*. (2014) empirically investigate flexibility/freedom of choice and political freedom and their combined effect on happiness. Their observation is that the preference for flexibility of choice (which comes close to their concept of “autonomy freedom”) and political freedom go together. In societies where there is political freedom, they find a distinct preference for a greater freedom of choice; whereas in societies used to limited political freedom, there is evidence of marked anxiety associated with greater freedom of choice. Their data suggest that overall, greater happiness correlates with greater freedom of choice, even in societies characterised by absence of political freedom. They use this observation as an argument against the “paradox of choice”, i.e., the claim that a situation where there is more to choose from may lead to choice overload, such as what Schwarz (2004) discusses. However, the kind of alternatives that they consider is very different from what Lipowski (1970), Schwarz (2004), Baucells and Sarin (2012), and we have in mind. Bavetta *et al*. (2014) focus primarily about alternatives being different political positions or parties that are on offer in elections where voters cast their votes to elect a winner. Their conclusion is that the availability of a vast spectrum of public/political opinion is overall preferred to a situation where there is less diversity of choice. We and other authors mentioned in this paper are concerned with the alternatives in question being material goods and services that individuals consume. These goods typically come under the category called “commodities”. We certainly do not have in mind political views as an alternative on offer to a decision maker. On the latter issue, although we have insufficient academic experience and knowledge, we would certainly like to believe the conclusions from Bavetta *et al*. (2014).

The problem that we see is how to integrate these two viewpoints with the concept of indirect utility function via separate axiomatisations. No problem arises in eliminating undesirable outcomes. Moreover, we make no compromise with the assumption that, at the second stage of the choice procedure, the decision maker will restrict his/her attention to the best alternatives available to him/her; and hence, the first-stage choice should be made while keeping this in mind. Hence, if the best alternatives of an opportunity set are better than the best alternatives of a second opportunity set, then our sophisticated decision maker unequivocally will choose the first opportunity set in the first stage of the decision making process. It is only when the decision maker is indifferent between the best alternatives of the two opportunity sets that problems concerning flexibility versus anxiety of choice arise. We call the two distinct preferences over opportunity sets that arise in these two situations as indirect utility reflecting anxiety of choice (IUA) and indirect utility reflecting flexibility of choice (IUF). We further note that Potoms and Lauwers (2013) refer to our IUF as Indirect Utility Freedom Rule. Hence, as may already have been anticipated, there is nothing particularly original about the IUF concept that this paper discusses. Whatever originality that may be associated with IUF is in its axiomatisation.

In this paper we consider six different properties. The first property is equivalent to the assertion that an opportunity set is just as good and no better as its set of best alternatives. Along with the second and third properties, we see that if the best elements of one opportunity set are better than the best elements of a second opportunity set, then the first opportunity set is preferred to the second opportunity set. The first, second, and fourth properties guarantee that if the best elements of the two opportunity sets have the same cardinality and are indifferent to each other, then the decision maker is indifferent between the two opportunity sets. At this point, the two roads leading to IUA and IUF diverge.

The fifth property says that any opportunity set whose elements are all equally good is less preferred to the opportunity set derived from the former by removing any one of its alternatives. These five axioms are necessary and sufficient for a transitive preference over opportunity sets to be an indirect utility function reflecting anxiety of choice.

The sixth property states that any opportunity set whose elements are all equally good is preferred to the opportunity set derived from the former by removing any one of its alternatives. The first four axioms along with this one are necessary and sufficient for a transitive preference over opportunity sets to be an indirect utility function that reflects flexibility of choice.

The difference between our axiomatisation and that of Potoms and Lauwers (2013) is that out of the five axioms that characterise IUF, the first four are satisfied by IUA as well. Potoms and Lauwers (2013) do not discuss anything similar to IUA. An interpretation of our results for IUF without comparing to the corresponding results for IUA (or the other way around) would defeat the purpose of our paper. Furthermore, while our axioms are entirely different from those of Potoms and Lauwers (2013), neither approach can dispense with invoking the set of the greatest elements of an opportunity set with respect to the relation generated on the set of alternatives by a preference over opportunity sets in their axiomatisations.

A considerable generalisation of IUF already exists in a paper by Puppe and Xu (2010). Puppe and Xu (2010) introduce the concept of a set of essential alternatives that is a non-empty subset of an opportunity set. The intended meaning of the set of essential alternatives is that of a revealed concept: an element is essential whenever the decision maker reveals it to be of value in connection with some menu. We hereby refer to the function that associates with each opportunity set its own set of essential alternatives as an essential choice function. Given an essential choice function, Puppe and Xu (2010) provide an axiomatic characterisation of those preferences over opportunity sets (which may not even be acyclic); such that one opportunity set is at least as good as another if and only if the cardinality of the set of essential elements made by the union of the two sets that belong to the first set is no less than the cardinality of the set of essential elements of the union of the two sets that belong to the second set. Once again, their axiomatisation, when restricted to the case of transitive preferences over opportunity sets and the set of essential alternatives being the best elements of the opportunity with respect to the relation that the preference over opportunity sets generates on the set of alternatives, is very different from our axiomatic characterisation. That their results hold even when preferences over opportunity sets may not even be acyclic is extremely useful and has wide applicability. Our axiomatic characterisation of IUF phrased in the language of Puppe and Xu (2010) could thus be viewed as providing a set of necessary and sufficient conditions for the set of essential alternatives to be the set of greatest/best alternatives of an opportunity set with respect to the relation generated on the set of alternatives by a transitive preference over opportunity sets.

The paper is arranged as follows. Section two presents the model as a standard approach to the problem. Section three defines the indirect utility function and its variants. Section four discusses the properties that are shared by both IUA and IUF. Section five provides an axiomatic characterisation of IUA. Section six gives an axiomatic characterisation of IUF. Section seven considers the relation between IUA and IUF and two axioms - Monotonicity and Concordance - which axiomatically characterise the indirect utility function. We show that IUA satisfies Concordance but not Monotonicity, whereas IUF satisfies Monotonicity but not Concordance. The final section concludes the discussion.

2 The Model

The genesis of the following model is in the seminal work of Kreps (1979), with subsequent contributions having been made by Lahiri (2003) among several others.

In what follows we shall use capital letters like A, B, etc. to denote sets of alternatives and small letters like x, y, etc. to denote alternatives.

Let X be a non-empty finite set of alternatives containing at least two elements. Let $\Psi(X)$ be the set of all non-empty subsets of X. Let $\Delta(X) = \{(x,x)/x\in X\}$ and $\Delta(\Psi(X)) = \{(A,A)/A\in\Psi(X)\}$. $\Delta(X)$ is called the diagonal of X, and $\Delta(\Psi(X))$ is called the diagonal of $\Psi(X)$.

A binary relation of R on X is said to be:

(a) **reflexive** if $\Delta(X) \subset R$;

(b) **complete** if given $x,y \in X$, with $x \neq y$, either $(x,y)\in R$ or $(y,x)\in R$;

(c) **transitive** if $\forall x,y,z \in X$: [$(x,y),(y,z)\in R$] implies [$(x,z)\in R$];

(d) an **ordering** if it is reflexive, complete, and transitive.

Sen (1970) presents the above terminology. What we refer as a complete binary relation is sometimes referred to as a “total” binary relation, as for instance in Richter (1971).

Given a binary relation of R on X, let $P(R) = \{(x,y) \in R \ / \ (y,x) \notin R\}$ denote the **asymmetric part** of R and let $I(R) = \{(x,y) \in R \ / \ (y,x) \in R\}$ denote the **symmetric part** of R.

Given an ordering of R on X and A $\in\Psi(X)$, let G(A,R) = $\{x\in A \,/(x,y) \in R$, whenever y $\in A\}$. G(A,R) denotes the set of **best elements** (or **alternatives**) with respect to R. The following is a well-known result for which citing any particular source would amount to discriminating against a multitude of others.

Proposition 1: Let R be an ordering on X and $A\in\Psi(X)$. Then, $G(A,R) \neq\phi$. For all $x,y \in G(A,R)$, we have $(x,y)\in I(R)$, and for all $x\in G(A,R)$ and $y\in X\backslash G(A,R)$, we have $(x,y)\in P(R)$.

A binary relation of $\Re$ on $\Psi(X)$ is said to be:
(a) **reflexive**, if $\Delta(\Psi(X)) \subset \Re$;
(b) **complete,** if $\forall A,B \in\Psi(X)$, with $A \neq B$, either $(A,B) \in\Re$ or $(B,A) \in\Re$;
(c) **transitive,** if $\forall A,B,C \in\Psi(X)$: $[(A,B),(B,C)\in\Re]$ implies $[(A,C)\in\Re]$.

Definition: A binary relation of $\Re$ on $\Psi(X)$ that is reflexive and complete is called a **Preference over Opportunity Sets** (POS). If in addition it is transitive, then it is called a **Transitive Preference over Opportunity Sets** (TPOS).

Given a binary relation of $\Re$ on $\Psi(X)$, let $P(\Re) = \{(A,B) \in\Re \,/\, (B,A) \notin\Re\}$ denote the **asymmetric part** of $\Re$ and let $I(\Re) = \{(A,B)\in\Re \,/\, (B,A) \in\Re\}$ denote the **symmetric part** of $\Re$.

Definition: Given a TPOS $\Re$, the **ordering generated by** $\Re$ (on X) and denoted $R(\Re)$ is the binary relation $\{(x,y)\in X\times X|(\{x\}, \{y\})\in\Re\}$.

Definition: A TPOS $\Re$ is said to be an **indirect utility function** (IU) if for all $A,B\in\Psi(X)$, $(A,B)\in\Re$ if and only if $G(A\cup B, R(\Re))\cap A\neq\phi$.

3 The Indirect Utility Function

We shall now present a proposition about IU that will facilitate further discussion.

Proposition 2: Let $\Re$ be an IU and A, where $B\in\Psi(X)$. The following two statements are then equivalent:

(i) $(A,B)\in\Re$.
(ii) For all $x\in G(A,R(\Re))$ and $y\in G(B,R(\Re))$ it is the case that $(x,y)\in R(\Re)$.

Proof: For the sake of completeness, we provide an easy proof of the proposition.

Let $(A,B)\in\Re$, $x\in G(A,R(\Re))$, and $y\in G(B,R(\Re))$. Since $\Re$ is an IU and $(A, B)\in\Re$, it is the case that $G(A\cup B, R(\Re))\cap A\neq\phi$. Let $z\in G(A\cup B, R(\Re))\cap A$. Since $z\in A$ and $x\in G(A,R(\Re))$, we have $(x,z)\in R(\Re)$. Since $z\in G(A\cup B, R(\Re))$ and $y\in B$, it is the case that $(z,y)\in R(\Re)$. By transitivity of $R(\Re)$, we get $(x,y)\in R(\Re)$. Thus, (i) implies (ii).

Now suppose that for all $x\in G(A,R(\Re))$ and $y\in G(B,R(\Re))$, it is the case that $(x,y)\in R(\Re)$. Let $z\in G(A,R(\Re))$ and $w\in G(B,R(\Re))$. Thus, $(z,x)\in R(\Re)$ for all $x\in A$ and $(w,y)\in R(\Re)$ for all $y\in B$. However, by hypothesis, $(z,w)\in R(\Re)$. Thus, by transitivity of $R(\Re)$, we get $(z,y)\in R(\Re)$ for all $y\in B$. Thus, $(z,x)\in R(\Re)$ for all $x\in A\cup B$, i.e., $z\in G(A\cup B, R(\Re))$. Thus, $z\in G(A\cup B, R(\Re))\cap A$ and so $G(A\cup B, R(\Re))\cap A\neq\phi$. Thus, $(A,B)\in\Re$, and (ii) implies (i). Q.E.D.

Lahiri (2003) offers the characterisation of an IU in terms of the following two properties.

Definition: A POS $\Re$ is said to satisfy **Concordance** if for all $A,B\in\Psi(X)$: $(A,B)\in\Re$ implies $(A, A\cup B)\in\Re$.

Definition: A POS $\Re$ is said to satisfy **Monotonicity** if for all $A,B\in\Psi(X)$: $B\subset A$ implies $(A,B)\in\Re$.

The following result is Theorem 1 in Lahiri (2003).

Theorem 1: A TPOS $\Re$ is an IU if and only if it satisfies Concordance and Monotonicity.

Notation: Given a set $A\in\Psi(X)$, let #A denote the cardinality of A.

Definition: A TPOS $\Re$ is said to be an **indirect utility function reflecting anxiety** (IUA) if given $A,B\in\Psi(X)$, $(A,B)\in\Re$ if **either** (i) $G(A\cup B,R(\Re))\cap B=\phi$; **or** (ii) $G(A\cup B,R(\Re))\cap A\neq\phi$, $G(A\cup B,R(\Re))\cap B\neq\phi$ & $\#G(A,R(\Re))\leq\#G(B,R(\Re))$.

Given that $\Re$ is TPOS, we can more elaborately rephrase the above definition as follows. A TPOS $\Re$ is said to be an **indirect utility function reflecting anxiety** (IUA) if given $A,B\in\Psi(X)$, $(A,B)\in\Re$ if **either** (i) there exists $x\in G(A,R(\Re))$ and $y\in G(B,R(\Re))$ such that $(\{x\},\{y\})\in P(\Re)$ (i.e. $(x,y)\in P(R(\Re))$); **or** (ii) for all

$x \in G(A,R(\Re))$ and $y \in G(B,R(\Re))$ it is the case that $(\{x\},\{y\}) \in I(\Re)$ & $\#G(A,R(\Re)) \leq \#G(B,R(\Re))$.

The concept of an indirect utility function reflecting anxiety tries to capture the inherent difficulties in arriving at a decision when there is an abundance of equally good competing decisions. It is not uncommon to see a selection committee meeting go on for hours, whereby a single candidate has to be chosen from a large list of "short-listed" candidates who are all considered to be equally good. The reason for such an extended negotiation may be (and usually is) that each member of the selection committee has his/her own favourite candidate and no one is apparently any more qualified than the other members on the list.

Definition: A TPOS $\Re$ is said to be an **indirect utility function reflecting flexibility** (IUF) if given $A,B \in \Psi(X)$, $(A,B) \in \Re$ if **either** (i) $G(A \cup B,R(\Re)) \cap B = \phi$; **or** (ii) $G(A \cup B,R(\Re)) \cap A \neq \phi$, $G(A \cup B,R(\Re)) \cap B \neq \phi$ & $\#G(A,R(\Re)) \geq \#G(B,R(\Re))$.

A large strand of the literature has discussed the concept of an indirect utility function reflecting freedom, and we indicate this once again in the following. It relates to what we like to believe about human behaviour and that we are conditioned to think that human beings always prefer to have more good things to choose from than less. Thus, we would normally prefer to visit a restaurant that offers a greater variety of palatable items on its menu versus another one that offers fewer such items. While this may indeed be the case when it comes to choosing between restaurants, this paper basically focuses on situations where a lesser number of good things is preferred over a greater amount of them.

Given that $\Re$ is TPOS, we can more elaborately rephrase the above definition as follows. A TPOS $\Re$ is said to be an **indirect utility function reflecting flexibility** (IUF) if given $A,B \in \Psi(X)$, $(A,B) \in \Re$ if **either** (i) there exists $x \in G(A,R(\Re))$ and $y \in G(B,R(\Re))$ such that $(\{x\},\{y\}) \in P(\Re)$ (i.e. $(x,y) \in P(R(\Re))$); **or** (ii) for all $x \in G(A,R(\Re))$ and $y \in G(B,R(\Re))$ it is the case that $(\{x\},\{y\}) \in I(\Re)$ & $\#G(A,R(\Re)) \geq \#G(B,R(\Re))$.

Note In the above definition, $(\{x\},\{y\}) \in I(\Re)$ can also be expressed as $(x,y) \in I(R(\Re))$.

In the sequel, the following class of sets (i.e. subset of $\Psi(X)$) will play a crucial role.

Definition: Given a TPOS $\Re$, let $\Psi^{\Re}(X) = \{A\in\Psi(X)|$. There does not exist any B that is a proper subset of A and $(B,A)\in I(\Re)\}$. A member of $\Psi^{\Re}(X)$ is said to be a **minimal set with respect to** $\Re$.

4 Some Properties of a TPOS

We will first discuss some properties of a TPOS that will be used in our axiomatisations.

Property 1: For all $A\in\Psi(X)$ and $x\in A\backslash G(A,R(\Re))$: $(A\backslash\{x\},A)\in I(\Re)$.

We can alternatively state this property as follows.
For all $A\in\Psi(X)$ and $x\in A$, if there exists $y\in A$ satisfying $(\{y\},\{x\})\in P(\Re)$, then $(A\backslash\{x\},A)\in I(\Re)$.
An immediate consequence of Property 1 is the following lemma.

Lemma 1: Let $\Re$ be a TPOS. Then $\Re$ satisfies Property 1 if and only if for all $A\in\Psi(X)$, it is the case that $(A, G(A,R(\Re)))\in I(\Re)$.

Proof: The proof that if $\Re$ is a TPOS then Property 1 implies $(A, G(A,R(\Re)))\in I(\Re)$ for all $A\in\Psi(X)$ is easy. Hence, suppose that for all $A\in\Psi(X)$, it is the case that $(A, G(A,R(\Re)))\in I(\Re)$. Let $A\in\Psi(X)$ and $x\in A\backslash G(A,R(\Re))$. Thus, $G(A\backslash\{x\},R(\Re)) = G(A,R(\Re))$ and by hypothesis $(A\backslash\{x\}, G(A\backslash\{x\},R(\Re)))\in I(\Re)$. Thus, by transitivity of $\Re$, we get $(A\backslash\{x\},A)\in I(\Re)$, i.e. Property 1 is satisfied. Q.E.D.

Lemma 2: Suppose $\Re$ is a TPOS satisfying Property 1 and let $A\in\Psi^{\Re}(X)$. Then for all $x,y\in A$, it is the case that $(\{x\}, \{y\})\in I(\Re)$ (i.e. $A = G(A,R(\Re))$).

Proof: Suppose $\Re$ is a TPOS satisfying Property 1 and let $A\in\Psi^{\Re}(X)$. For a contradiction, suppose there exists $x,y\in A$ such that $(\{x\},\{y\})\notin I(\Re)$. Then either $(\{x\},\{y\})\in P(\Re)$ or $(\{y\}, \{x\})\in P(\Re)$. Either way, $G(A,R(\Re))$ is a proper subset of A. By Property 1, $(G(A,R(\Re)), A)\in I(\Re)$. This contradicts the assumption that $A\in\Psi^{\Re}(X)$ and proves the lemma. Q.E.D.

Lemma 2 says is that if $\Re$ is a TPOS satisfying Property 1, then $\Psi^{\Re}(X) \subset \{A\in\Psi(A)|$ $A = G(A,R(\Re))\}$. The next property allows us to establish the converse inclusion.

Property 2: Let $A,B \in \Psi(X)$ with $B \subset\subset A$ and $(A,B) \in I(\Re)$. Then there exists $x,y \in A$ such that $(\{x\},\{y\}) \notin I(\Re)$.

Lemma 3: Suppose $\Re$ is a TPOS satisfying Property 2. Then, $\{A \in \Psi(A) | \ A = G(A,R(\Re))\} \subset \Psi^{\Re}(X)$.

Proof: Suppose $\Re$ is a TPOS satisfying Property 2 and let $A \in \Psi(A)$ satisfy $A = G(A,R(\Re))$. Thus, by Proposition 1, for all $x,y \in A$, it is the case that $(x,y) \in I(R(\Re))$ (i.e. $(\{x\},\{y\}) \in I(\Re)$). For a contradiction, suppose $A \notin \Psi^{\Re}(X)$. Thus, there exists a non-empty proper subset B of A such that $(A,B) \in I(\Re)$. By Property 2, there exists $x,y \in A$ such that $(\{x\},\{y\}) \notin I(\Re)$, leading to a contradiction. Thus, $\{A \in \Psi(A) | \ A = G(A,R(\Re))\} \subset \Psi^{\Re}(X)$. Q.E.D.

In view of Lemmas 2 and 3, we have the following proposition.

Proposition 3: Suppose $\Re$ is a TPOS satisfying Properties 1 and 2; then $\Psi^{\Re}(X) = \{A \in \Psi(A) | \ A = G(A,R(\Re))\}$.

Note: In $\Psi^{\Re}(X) \times \Psi^{\Re}(X)$, there are two types of pairs of opportunity sets. First, there are the pairs (A,B) such that their union belongs to $\Psi^{\Re}(X)$; and then there are those pairs whose union do not belong to $\Psi^{\Re}(X)$. Let us consider the latter pairs first.

Property 3: Let A, B $\in \Psi^{\Re}(X)$ be such that $A \cup B \notin \Psi^{\Re}(X)$. If $(B,A) \in \Re$, then $(A \cup B,A) \in P(\Re)$.

As a consequence of Properties 1, 2, and 3, we obtain the following proposition.

Proposition 4: (a) Let $\Re$ be either an IUA or an IUF; then $\Re$ satisfies Properties 1, 2, and 3. (b) Let $\Re$ be a TPOS satisfying Properties 1, 2, and 3 and let A, B $\in \Psi^{\Re}(X)$ be such that $A \cup B \notin \Psi^{\Re}(X)$; then, either $(A,B) \in P(\Re)$ or $(B,A) \in P(\Re)$. Furthermore, $(A,B) \in P(\Re)$ if and only if $G(A \cup B, R(\Re)) = A$(i.e. for all $x \in A$ and $y \in B$, it is the case that $(x,y) \in P(R(\Re))$.

Proof: The proof of (a) is easy and so let us prove (b).

Let $\Re$ be a TPOS satisfying Properties 1, 2, and 3 and let A, B $\in \Psi^{\Re}(X)$ be such that $A \cup B \notin \Psi^{\Re}(X)$. By Proposition 3, all alternatives in A are indifferent to each other, and all alternatives in B are indifferent to each other. Again, by Proposition 3,

$A \cup B \notin \Psi^{\Re}(X)$ implies that all elements in $A \cup B$ are not indifferent to each other. Thus, $A \cap B = \phi$, and A and B are in two separate indifference classes. Thus, either each element in A is preferred to each element in B or each element in B is preferred to each element in A. Thus, either $G(A \cup B, R(\Re)) = A$ or $G(A \cup B, R(\Re)) = B$.

Without loss of generality, suppose each element in A is preferred to each element in B such that $G(A \cup B, R(\Re)) = A$. For a contradiction, suppose $(B,A) \in \Re$. By Property 3, $(A \cup B, A) \in P(\Re)$. However, by Property 1, $(A \cup B, G(A \cup B, R(\Re))) \in I(\Re)$, i.e. $(A \cup B, A) \in I(\Re)$, leading to a contradiction. Thus, $(A,B) \in P(\Re)$. Similarly, if $G(A \cup B, R(\Re)) = B$, then $(B,A) \in P(\Re)$. Since $G(A \cup B, R(\Re)) \in \{A,B\}$, it follows that $(A,B) \in P(\Re)$ if and only if $G(A \cup B, R(\Re)) = A$. Q.E.D.

This leaves us with those pairs $(A,B) \in \Psi^{\Re}(X) \times \Psi^{\Re}(X)$ such that $A \cup B \in \Psi^{\Re}(X)$.

Property 4: Let A, B $\in \Psi^{\Re}(X)$ be such that $A \cup B \in \Psi^{\Re}(X)$. Let $x \in A$ and $y \in B$. Then $(A,B) \in I(\Re)$ if and only if $(A \setminus \{x\}, B \setminus \{y\}) \in I(\Re)$.
The following proposition is an easy consequence of Properties 1, 2, and 4.

Proposition 5: (a) Let $\Re$ be either an IUA or an IUF. Thus, $\Re$ satisfies Property 4. (b) Let $\Re$ be a TPOS satisfying Properties 1, 2, and 4 and let A, B $\in \Psi^{\Re}(X)$ be such that $A \cup B \in \Psi^{\Re}(X)$ and #A = #B. Then $(A,B) \in I(\Re)$.

Proof: (a) is easy to prove, and so let us prove (b). $A \cup B \in \Psi^{\Re}(X)$ implies all elements of $A \cup B$ are indifferent to each other. Thus, let $A = \{x_1,\ldots,x_K\}$ and $B = \{y_1, y_2,\ldots, y_K\}$. Clearly, $\{x_1, x_2\} \cup \{y_1, y_2\} \in \Psi^{\Re}(X)$. Furthermore , $(\{x_1\},\{y_1\}) \in I(\Re)$. By Property 4, $(\{x_1, x_2\},\{y_1, y_2\}) \in I(\Re)$. Suppose $(\{x_1,\ldots, x_k\},\{y_1,\ldots, y_k\}) \in I(\Re)$ for some $k \in \{1,\ldots,K-1\}$. Clearly $\{x_1,\ldots, x_{k+1}\} \cup \{y_1,\ldots, y_{k+1}\} \in \Psi^{\Re}(X)$. Hence, by Property 4, $(\{x_1,\ldots, x_k, x_{k+1}\},\{y_1,\ldots, y_k, y_{k+1}\}) \in I(\Re)$. Thus, by a standard induction argument we get $(A,B) \in I(\Re)$. Q.E.D.

Note: For Proposition 5, we could have used the following weaker version of Property 4 and come to the same conclusion.

Let A, $B \in \Psi^{\Re}(X)$ be such that $A \cup B \in \Psi^{\Re}(X)$. Let $x \in A$ and $y \in B$. Then $(A,B) \in I(\Re)$ if $(A \setminus \{x\}, B \setminus \{y\}) \in I(\Re)$.

5 Anxiety of Choice

We now want to consider the situation where given two opportunity sets of different sizes, such that all elements of their union are indifferent to each other, the smaller opportunity set is preferred to the larger opportunity set. The simplest property, which in the presence of Properties 1, 2, and 4 guarantees this conclusion, is the following.

Definition: A TPOS is said to satisfy **anxiety of choice** if for all $A \in \Psi^{\Re}(X)$ with $\#A \geq 2$, if $x \in A$, then $(A \setminus \{x\}, A) \in P(\Re)$.
The above could be rephrased as Property 5.

Property 5: For all $A \in \Psi^{\Re}(X)$ with $\#A \geq 2$, if $x \in A$, then $(A \setminus \{x\}, A) \in P(\Re)$.
Since this a crucial property and plays the defining role for IUA, we shall refer to it as "anxiety of choice" instead of Property 5.
Anxiety of choice is what we would expect to observe when there is overabundance.

Proposition 6: (a) Let $\Re$ be an IUA; then $\Re$ satisfies anxiety of choice. (b) Let $\Re$ be a TPOS satisfying Properties 1, 2, and 4 and anxiety of choice. Let $A,B \in \Psi^{\Re}(X)$ be such that $A \cup B \in \Psi^{\Re}(X)$ and $\#A < \#B$. Then $(A,B) \in P(\Re)$.

Proof: (a) is easy to prove, so let us prove (b). Thus, let $\Re$ be a TPOS satisfying Properties 1, 2, and 4 and anxiety of choice. Let $A,B \in \Psi^{\Re}(X)$ be such that $A \cup B \in \Psi^{\Re}(X)$ and $\#A < \#B$. Suppose $\#B - \#A = K$. There then exists a non-empty subset $\{x_1, x_2, \ldots, x_K\}$of $B \setminus A$. Let $\widehat{A} = A \cup \{x_1, x_2, \ldots, x_K\}$. Thus, $\widehat{A} \cup B \in \Psi^{\Re}(X)$ and $\#\widehat{A} = \#B$. Hence, by Proposition 5, $(\widehat{A},B) \in I(\Re)$.

By repeated application of the anxiety of choice property, we get $(A, \widehat{A}) \in P(\Re)$. Thus, by transitivity of $\Re$, we get $(A,B) \in P(\Re)$. Q.E.D.
We can now combine Propositions 4, 5, and 6 to obtain the following theorem.
Theorem 2: Let $\Re$ be a TPOS. Then $\Re$ is an IUA if and only if $\Re$ satisfies Properties 1, 2, 3, and 4 and the anxiety of choice property.

6 Flexibility of Choice

This section targets the exact opposite of anxiety of choice, i.e. flexibility of choice. We thus want to consider the situation where given two opportunity sets of different sizes, such that all elements of their union are indifferent to each other, the larger opportunity set is preferred to the smaller opportunity set. Towards that end, we introduce the following property.

Definition: A TPOS is said to satisfy **flexibility of choice** if for all $A \in \Psi^{\Re}(X)$ with $\#A \geq 2$, if $x \in A$, then $(A, A \setminus \{x\}) \in P(\Re)$.

The above could be rephrased as Property 6.

Property 6: For all $A \in \Psi^{\Re}(X)$ with $\#A \geq 2$, if $x \in A$, then $(A, A \setminus \{x\}) \in P(\Re)$.

Once again, since this is a crucial property and plays the defining role for IUF, we shall refer to it as "flexibility of choice" instead of Property 6. Flexibility of choice is what we would expect to observe when there is relative deprivation or when the issue concerns our rights and freedoms.

We will skip the proof of the following proposition since it is analogous to the proof of Proposition 6.

Proposition 7: (a) Let $\Re$ be an IUF; then $\Re$ satisfies flexibility of choice. (b) Let $\Re$ be a TPOS satisfying Properties 1, 2, and 4 and flexibility of choice. Let $A,B \in \Psi^{\Re}(X)$ be such that $A \cup B \in \Psi^{\Re}(X)$ and $\#A > \#B$. Then $(A,B) \in P(\Re)$.

We may now combine Propositions 4, 5, and 7 to obtain the following theorem.

Theorem 3: Let $\Re$ be a TPOS. Then $\Re$ is an IUF if and only if $\Re$ satisfies Properties 1, 2, 3, and 4 and the flexibility of choice property.

7 Relationship with Concordance and Monotonicity

The following result establishes the relationship of our proposed binary relations with Concordance and Monotonicity.

Proposition 8: (a) Let $\Re$ be an IUA. Then it satisfies Concordance, but does not satisfy Monotonicity. (b) Let $\Re$ be an IUF. Then it satisfies Monotonicity, but does not satisfy Concordance.

Proof: (a) Let $\Re$ be an IUA and suppose $(A,B) \in \Re$. Thus, by Lemma 1, $(G(A,R(\Re)),G(B,R(\Re))) \in \Re$.

If $x \in G(A,R(\Re))$ and $y \in G(B,R(\Re))$ implies $(x,y) \in I(R(\Re))$, then $G(A \cup B,R(\Re)) = G(A,R(\Re)) \cup G(B,R(\Re))$ and $(x,z) \in I(R(\Re))$ for all $x \in G(A,R(\Re))$ and $z \in G(A \cup B,R(\Re))$. Since $\# G(A,R(\Re)) \leq \# G(A \cup B,R(\Re)) = \# G(A,R(\Re)) \cup G(B,R(\Re))$, and since $\Re$ is an IUA, we get $(A,A \cup B) \in \Re$.

If $x \in G(A,R(\Re))$ and $y \in G(B,R(\Re))$ implies $(x,y) \in P(R(\Re))$, then $G(A \cup B,R(\Re)) = G(A,R(\Re))$ and so $(A,A \cup B) \in I(\Re) \subset \Re$.

Thus, $\Re$ satisfies Concordance.

To show that $\Re$ does not satisfy Monotonicity, let $X = \{x,y\}$ and suppose $(x,y) \in I(R(\Re))$. Thus, $(\{x\}, \{x,y\}) \in P(\Re)$, contradicting Monotonicity.
(b) Let $\Re$ be an IUF and suppose $B \subset A$. If $G(A,R(\Re)) \cap G(B,R(\Re)) \neq \phi$, then in fact $G(B,R(\Re)) \subset G(A,R(\Re))$ and so $(A,B) \in \Re$. If $G(A,R(\Re)) \cap G(B,R(\Re)) = \phi$, then $x \in G(A,R(\Re))$ and $y \in G(B,R(\Re))$ implies $(x,y) \in P(R(\Re))$ and so $(A,B) \in P(\Re) \subset \Re$. Thus, $\Re$ satisfies Monotonicity.
To show that $\Re$ does not satisfy Concordance, let $X = \{x,y\}$ and suppose $(x,y) \in I(R(\Re))$. Thus, $(\{x\}, \{y\}) \in I(\Re) \subset \Re$ and yet $(\{x,y\}, \{x\}) \in P(\Re)$. This contradicts Concordance. Q.E.D.
Note: It is tempting to confuse IUF with the following property.
Property*: For all $A,B \in \Psi(X)$, $(A,B) \in \Re$ if and only if $\#G(A, R(\Re)) \geq \#G(A, R(\Re))$.
A simple example as follows illustrates that the two concepts are different and that if $\Re$ satisfies Property *, then it does not satisfy Monotonicity.
Example: Let $X = \{x,y,z\}$ and $\Re$ be any TPOS satisfying $(\{x\},\{y\}) \in I(\Re)$, $(\{z\},\{x\}) \in P(\Re)$ and $(\{z\},\{y\}) \in P(\Re)$. Thus, $R(\Re) = \Delta(X) \cup \{(x,y), (y,x), (z,x), (z,y)\}$. Let $A = X$ and $B = \{x,y\}$. Thus, $G(X, R(\Re)) = \{z\}$ and $G(X, R(\Re)) = \{x,y\}$. If $\Re$ were to satisfy Property*, then we get $(\{x,y\}, X) \in P(\Re)$ leading to a violation of Monotonicity. However, since $(z,x), (z,y) \in P(R(\Re))$, if $\Re$ is an IUF, then we get $(X,\{x,y\}) \in P(\Re)$ and Monotonicity is no longer violated. This shows that IUF and Property* are conceptually very different.

8 Conclusion

"More is better" is a principle that many economic consumer behaviour theories assume, and a similar question can be raised about IUA. Why should a decision maker prefer fewer versus more alternatives to choose from if all alternatives are equally good?

There are two answers to this question that we are aware of. The first is the empirical evidence that has been gathered by psychologists and which has been discussed in the introduction. The second is the implicit or explicit (psychological) cost of making comparisons. Unless the decision maker is blind folded and asked to pick an alternative from those that are all equally good, there is always a cost involved in the act of choice (even if it is plain and simple "randomization") that choice theory does not explicitly account for. One has to use one's mind (or "hear an inner voice" as some would like to believe), and this requires some effort, however small that may be. Lipowski (1970), Schwarz (2004), Baucells and Sarin (2012), and others note this, and it is partially accounted for in IUA. In a way, this paper is really about IUA and IUF being a by-product of our analysis. Extending IUA to preferences over opportunity sets that are not transitive may well be the subject matter of future research.

Acknowledgment

I would like to thank Prasanta Pattanaik for views and observations at various times, which have lead to my interest in this paper's topic. I would like to thank Clemens Puppe for suggesting references, which have lead to a significant improvement in this paper. I would also like to thank Sebastiano Baveta and Matthew Ryan for helpful discussions. An earlier version of this paper was presented at X GRASS Workshop held at NYU (Florence) on September 20 and 21, 2016. I would like to thank Ernesto Savaglio for detailed comments and concerns, which I have tried to address in this paper. I would also like to thank Antonio Nicolo for observations on the paper during the workshop. I would like to put on record my immense gratitude to two anonymous referees of this journal for their valuable comments and suggestions for its improvement. Finally, as always, I remain grateful to Professor Dipankar Dasgupta for encouragement and advice all along the way.

References

Arrow, K. J., (1994), "A Note on Freedom and Flexibility," In: Basu, K., P. K. Pattanaik and K. Suzumura (eds.), *Choice, Welfare, and Development*, A Festschrift in Honor of Amartya K. Sen, Clarendon Press, Oxford.

Baucells, M. and R. Sarin, (2012), *Engineering Happiness: A New Approach for Building a Joyful Life*, University of California Press, Oakland CA.

Bavetta, S., D. Maimone, P. Miller, and P. Navarra, (2014), *Autonomy, Political Freedom and Happiness (mimeo)*.

Gekker, R., (2001), "On the Axiomatic Approach to Freedom as Opportunity: A General Characterization Result," *Math Soc Sci*, 42, 169-177.

Klemisch-Ahlert, M., (1993), "Freedom of Choice: A Comparison of Different Rankings of Opportunity Sets," *Soc Choice Welfare*, 10, 189-207.

Konnikova, M., (2014), *When It's Bad to Have Good Choices*, The New Yorker (August 1, 2014).

Koopmans, T. C., (1964), "On the Flexibility of Future Preferences," In: Shelley, M. W. and G. L. Bryan (eds.), *Human Judgments and Optimality*, Wiley, New York.

Kreps, D. M., (1979), "A Representation Theorem for 'Preference for Flexibility'," *Econometricam*, 47, 565-577.

Kreps, D. M., (1988), *Notes on the Theory of Choice (Underground Classics in Economics)*, Westview Press, Boulder, London.

Kreps, D. M., (1990), *A Course in Microeconomic Theory*, Princeton University Press, Princeton, NJ.

Lahiri, S., (2003), "Justifiable Preferences Over Opportunity Sets," *Social Choice and Welfare*, 21, 117-129.

Lipowski, Z. J., (1970), "The Conflict of Buridan's Ass or Some Dilemmas of Affluence: The Theory of Attractive Stimulus Overload," *American Journal of Psychiatry*, 127, 273-279.

Malishevsky, A. V., (1997), "An Axiomatic Justification of Scalar Optimization," In: Tangian, A. and J. Gruber (eds.), *Constructing Scalar Valued Objective Functions (Lecture Notes in Economics and Mathematical Systems) Springer*, Berlin Heidelberg New York.

Pattanaik, P. K. and Y. Xu, (1990), "On Ranking Opportunity Sets in Terms of Freedom of Choice," *Rech Econ Louvain*, 56, 383-390.

Pattanaik, P. K. and Y. Xu, (1997), *On Diversity and Freedom of Choice (mimeo)*.

Pattanaik, P. K. and Y. Xu, (1998), "On Preference and Freedom," *Theory Decision*, 44, 173-198.

Potoms, T. and L. Lauwers, (2013), "Ranking Opportunity Sets, Averaging versus Preference for Freedom," Centre for Economic Studies, KU Leuven, Discussion Paper Series, DPS 13.14.

Puppe, C., (1995), "Freedom of Choice and Rational Decisions," *Soc Choice Welfare*, 12, 137-153.

Puppe, C., (1996), "An Axiomatic Approach to 'Preference for Freedom of Choice'," *J Econ Theory*, 68, 174-199.

Puppe, C. and Y. Xu, (2010), "Essential Alternatives and Freedom Rankings," *Soc Choice Welfare*, 33, 669-685.

Qin, D., (2015), "On Justifiable Preferences Over Opportunity Sets," *Soc Choice Welfare*, 45, 269-285.

Richter, M. K., (1971), "Rational Choice," In: Chipman, J. S., L. Hurwicz, M. K. Richter and H. F. Sonnenschein (eds.), *Preferences, Utility, and Demand: A Minnesota Symposium,* New York: Harcourt, Brace, Jovanovich, Chapter 2, 29-58.

Ryan, M., (2014), "Path Independent Choice and Ranking of Opportunity Sets," *Soc Choice Welfare*, 42, 193-213.

Schwartz, B., (2004), *The Paradox of Choice: Why More is Less*, Harper Perennial, NY.

Sen, A. K., (1970), *Collective Choice and Social Welfare*, Holden Day, San Francisco.

Sen, A. K., (1988), "Freedom of Choice: Concept and Content," *Europ Econ Rev*, 32, 269-294.

Sen, A. K., (1990), "Welfare, Freedom and Social Choice: A Reply," *Rech Econ Louvain*, 56, 451-485.

Sen, A. K., (1991), "Welfare, Preference and Freedom," *J Econ*, 50, 15-29.