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A Quantile Regression Approach to the Multiple Period Value at Risk Estimation

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This research focuses on methods for multiple period Value at Risk (VaR) estimation by utilizing some common approaches like RiskMetrics and empirical distribution and examining quantile regression. In a simulation study we compare the least square and quantile regression percentiles with the actual percentiles for different error distributions. We also discuss the method of selecting response and explanatory variables for the quantile regression approach. In an empirical study, we apply the three VaR estimation approaches to the aggregate returns of four major market indices. The results indicate that the quantile regression approach is better than the other two approaches.

Keywords: quantile regression, value at risk, risk measures
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1 Introduction

Value at Risk (VaR) has been promoted by regulatory bodies and used by financial institutions to measure the market risk in a portfolio of financial assets. This risk management tool measures the minimum loss of a portfolio over some time horizon

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for a certain probability, say 1% or 5%, presenting how bad the investment can be in time of adversity. Strictly speaking, finding VaR is equivalent to estimating the quantile of a return distribution. The application of VaR is not only a risk measure, but is also a form of risk control and active risk management (see Jorion (2006)).

VaR is used to determine the amount of capital that a bank is required to maintain to reflect the market risks it is bearing. Financial institutions are required to keep capital at least amounted to 8%, known as the minimum capital adequacy ratio (CAR), of the total risk-weighted exposures that cover credit risk, market risk, and operational risk (see Basel (2001)):

$$\begin{array}{ccc} \textit{Total} & & \\ \textit{Required Capital} & \geq & 8\% \times \textit{ Total Risk Weighted Exposures} \\ & = & \begin{array}{c}\textit{Sum of Capital Charge for} \\ \textit{Credit Risk, Market Risk \& Operational Risk.}\end{array} \end{array}$$

VaR is mainly related to the market risk capital charge. In Hong Kong, aside from the standardized approach and the internal models approach stated in the revised Basel Capital Accord (Basel (1996a), Basel (1996b)), the European Union Capital Adequacy Directive may be used to measure the market risk capital charge (see Hong Kong (1997)). Many international financial institutions have built internal models and quantified the market risk of the underlying trading portfolio based on the VaR approach. Under the specification of the Basel Committee, VaR is the loss over a 10-day period that is expected to happen only 1% of the time, and an observation window of at least 1 year is used to compute it. In brief, if there are no credit risk and operational risk, then the minimum required capital is simply the market risk capital charge.

The Basel Accord allows banks to calculate their 10-day VaR using a 1-day VaR number scaled up to 10-day by the square root of time rule. Two assumptions are made: 1) the variance of the 10-day return is simply ten times that of the daily return; and 2) the 1-period return and 10-period return have the same distribution. However, it is well known that the volatility of daily returns in financial markets is not constant over time and the distributions for daily returns and multiple period returns may be different, and so this estimation method for multiple period VaR is not appropriate.

Some common approaches estimating VaR used by practitioners include parametric, historical, and Monte-Carlo simulations (see Jorion (1997)). RiskMetrics (Morgan and Reuters (1996), Mina and Xiao (2001)) is a very popular parametric method calculating multiple period VaR and is an IGARCH(1,1) model. The RiskMetrics model is easy to use, because of the two assumptions made: 1) volatility is estimated using the exponentially weighted moving average (EWMA) approach; and 2) the daily returns are normally distributed. The square root of time rule for multiple period volatility used in RiskMetrics is shown to be valid by Wong and So (2003). However, the normality assumption may not be sound in real financial markets (see Duffie and Pan (1997)). It is found that the returns in financial markets generally have heavier tails than normal distributions, and so the RiskMetrics estimate exhibits bias especially when we consider the 1% chance case. The conditional distribution for multiple period returns in financial markets is also very complicated, because single period returns are not independent and identically distributed.

In the historical method, VaR can be obtained from the empirical quantile of the portfolio returns by assuming the returns have a fixed distribution. One drawback of this method is that testing the sensitivity of VaR for different assumptions is not flexible. Other VaR estimation methods are proposed - for example, the extreme value approach (see Ho *et al*. (2000)). Chen *et al*. (2012) consider a range of parametric volatility models on 1-day and 10-day forecasts of the 1% VaR. Their results reveal that GARCH models outperform stochastic volatility models in almost all cases, and all models forecast VaR less accurately and anti-conservatively post global financial crisis.

This study compares three VaR estimation methods: RiskMetrics, empirical distribution, and quantile regression. The empirical distribution approach can be seen as a modified historical approach. VaR is obtained from the empirical distribution of returns standardized by the RiskMetrics conditional volatility. The quantile regression approach is a distribution free method that is used to estimate VaR (see Chernozhukov and Umantsev (2001), Engle and Manganelli (2004), Taylor (1999), Taylor (2008), Gerlach *et al*. (2011)).

Researchers find quantile regression easy to apply, because of its similar nature to traditional least square regression. We produce quantile models that are functions of variables such as the conditional volatilities and the length of the holding period, as suggested by the GARCH-derived variance expressions. The approach in Taylor (2008) is different from ours as it uses nonparametric kernel smoothing in obtaining the quantiles. Gerlach *et al*. (2011) develop a Bayesian estimator for the nonlinear conditional autoregressive VaR (CAViaR) model family.

This article is structured as follows. Section 2 gives a brief introduction to the quantile regression. Section 3 contains a simulation study that compares the least square and quantile regression percentiles with the actual percentiles for different error distributions. Section 4 provides the details of the VaR estimation methods being used and the methods to evaluate the VaR estimates. Section 5 presents an empirical application to real data where we compare the VaR estimates calculated under different approaches by using data on four stock market indices. This section also discusses the method of selecting response and explanatory variables for the quantile regression approach. Section 6 concludes this article.

2 Quantile Regression

Regression analysis examines the relationship between two or more quantitative variables so that one variable can be predicted from the other variable(s). The conditional mean of the response variable given the predictor variable(s) can be studied in classical least square (mean) regression analysis. Researchers have long been interested in studying similar conditional statistics that measure relative standing, e.g. median and quantile, and have proposed various alternatives to least square regression (see Hogg (1975), Hogg and Randles (1975)).

Koenker and Bassett (1978, 1982) introduce a new class of estimator: quantile regression. Quantile regression can be seen as a complement of least square regression. For an analogy between mean and quantile, one can see the summary written by Koenker and Hallock (2001).

Suppose y is the response variable and $x_{1,}...,x_k$ are the predictor variables, where k is the number of predictor variables. Assume that the first component $\mathbf{x}_0$ is a column of 1s, and the column vectors $\mathbf{x}_{1,}...,\mathbf{x}_k$ are the n observations on variables $x_{1,}...,x_k$, respectively. The data can be formed in an $n\times(k+1)$ data matrix $\mathbf{X}$. We will also let $\mathbf{y}$ be the vector of the n observations $y_{1,}...,y_n$.

$$\begin{array}{lcccc}
\mathbf{y} & \mathbf{x}_0 & \mathbf{x}_1 & \cdots & \mathbf{x}_k \\
y_1 & 1 & x_{11} & \cdots & x_{k1} \\
\vdots & \vdots & \vdots & & \vdots \\
y_n & 1 & x_{1n} & \cdots & x_{kn}
\end{array}$$

Let $\boldsymbol{\epsilon}$ be the column vector containing the n random errors $\epsilon_1, \cdots, \epsilon_n$. Here, y_i can be expressed as:

$$y_i = \mathbf{x}_i'\boldsymbol{\beta} + \epsilon_i, \quad i = 1, \ldots., n,$$

where $\mathbf{x}_i$ is the transpose of the i^{th} row of $\mathbf{X}$, and $\boldsymbol{\beta}$ is the parameter vector. The conditional mean of y, $\mathbf{E}[y|\mathbf{x}_i] = \mathbf{x}_i'\boldsymbol{\beta}$, can be obtained by solving:

$$\min_{\beta} \sum_{i=1}^{n} (y_i - \mathbf{x}_i'\boldsymbol{\beta})^2 \,,$$

and thus $\hat{y}_i = \mathbf{x}_i'\hat{\boldsymbol{\beta}}$, the fitted value of y, is the estimate of $\mathbf{E}[y|\mathbf{x}_i]$.

Using a similar idea, we now consider a quantile. Suppose the distribution function of y is $F(y) = P(Y \le y)$, and then the θ^{th} quantile of y is defined as:

$$Q(\theta) = \inf\{y : F(y) \ge \theta\} \,,$$

for any $0 < \theta < 1$. Its estimate is denoted by $\hat{Q}(\theta)$.

According to Koenker and Bassett (1978), the θ^{th} conditional quantile function of a variable y given covariates of $\mathbf{x}_i$:

$$\hat{Q}(\theta|\mathbf{x}_i) = \mathbf{x}_i'\hat{\boldsymbol{\beta}}(\theta),$$

where $\hat{\boldsymbol{\beta}}(\theta)$ is a vector of estimated parameters dependent on θ and the estimate $\hat{Q}(\theta|\mathbf{x}_i) = \mathbf{x}_i'\hat{\boldsymbol{\beta}}(\theta)$, can be found by:

$$v_1 = \min_{\boldsymbol{\beta}} \left[\sum_{y \in \{i: y_i \ge \mathbf{x}_i'\boldsymbol{\beta}\}} \theta |y_i - \mathbf{x}_i'\boldsymbol{\beta}| + \sum_{y \in \{i: y_i < \mathbf{x}_i'\boldsymbol{\beta}\}} (1-\theta)|y_i - \mathbf{x}_i'\boldsymbol{\beta}| \right]. \qquad (1)$$

We note that $\hat{\boldsymbol{\beta}}(\theta)$ varies with θ. Even for the same dataset, it is very likely the values of $\hat{\boldsymbol{\beta}}(\theta)$ will be different for different θs.

Compared with least square regression, quantile regression has less restriction as it does not assume any specific distribution for the random error $\boldsymbol{\epsilon}$. Conditional quantiles can be examined at different values of θ. Quantile regression has been used in many financial and social science studies (see Bassett and Chen (2001), Bassett *et al*. (2002), Chernozhukov and Umantsev (2001), Taylor (1999), Taylor and Bunn (1999)). As only the conditional quantile will be discussed, we shall simplify $Q(\theta|\mathbf{x}_i)$ as $Q_i(\theta)$ afterwards.

3 Simulation

As mentioned in Section 2, least square regression performs poorly if the error distribution assumptions do not hold, whereas no specific error distribution is required for quantile regression. This section compares the least square and quantile regression percentiles with the actual percentiles for different error distributions. The definitions for different percentiles, the design of the simulation study, and the results are provided in the following sub-sections.

3.1 Percentile Estimation

Suppose we have n observations $y_{1,} y_{2,} ..., y_n$, which is a linear combination of some covariates $\mathbf{x}_i$ and ϵ_i:

$$y_i = \mathbf{x}_i' \boldsymbol{\beta} + \epsilon_i, \quad \epsilon_i \sim D(0, \sigma^2), \tag{2}$$

where $D(0, \sigma^2)$ denotes a distribution with mean 0 and variance σ^2. The actual percentile, $Q_i(\theta)$, for observation i at the θ^{th} quantile is:

$$Q_i(\theta) = \mathbf{x}_i' \boldsymbol{\beta} + q(\theta),$$

where $q(\theta)$ is the inverse of the distribution function when the probability equals θ. If the random variable is a standard normal distribution, then $q(0.025) = -1.96$.

For a quantile estimation using the least square regression approach, random errors are assumed to follow a normal distribution. Suppose that the variance of y is known and the parameters $\boldsymbol{\beta}$ are estimated by the least square method. The estimated least square percentile, $\hat{Q}_{LS,i}(\theta)$, is therefore:

$$
\begin{aligned}
\hat{Q}_{LS,i}(\theta) &= inf\left\{y: \hat{F}(y|\mathbf{x}_i) \geq \theta\right\} \\
&= inf\left\{y: \Phi\left(\frac{y-\mathbf{x}_i'\hat{\boldsymbol{\beta}}}{\sigma}\right) \geq \theta\right\} \\
&= inf\left\{y: \frac{y-\mathbf{x}_i'\hat{\boldsymbol{\beta}}}{\sigma} \geq \Phi^{-1}(\theta)\right\} \\
&= inf\left\{y: y \geq \mathbf{x}_i'\hat{\boldsymbol{\beta}} + \sigma\Phi^{-1}(\theta)\right\} \\
&= \mathbf{x}_i'\hat{\boldsymbol{\beta}} + \sigma\Phi^{-1}(\theta),
\end{aligned}
$$

where $\hat{F}(y|\mathbf{x}_i)$ is the conditional distribution function of y given $\mathbf{x}_i$, $\hat{\boldsymbol{\beta}}$ is the vector of least square regression coefficients, and $\Phi(\theta)$ is the cumulative distribution function of a standard normal random variable.

The estimated quantile regression percentile, $\hat{Q}_{QR,i}(\theta)$, for observation i at the θ^{th} quantile is:

$$\hat{Q}_{QR,i}(\theta) = \mathbf{x}_i'\hat{\boldsymbol{\beta}}(\theta),$$

where $\hat{\boldsymbol{\beta}}(\theta)$ is the vector of quantile regression coefficients. We define the error of estimating percentile = actual percentile - estimated percentile. In the least square regression approach, the estimation error is:

$$
\begin{aligned}
& Q_i(\theta) - \hat{Q}_{LS,i}(\theta) \\
= \quad & \mathbf{x}_i'(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}) + q(\theta) - \sigma\Phi^{-1}(\theta),
\end{aligned}
$$

and in the quantile regression approach, the estimation error is:

$$
\begin{aligned}
& Q_i(\theta) - \hat{Q}_{QR,i}(\theta) \\
= \quad & \mathbf{x}_i'(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}(\theta)) + q(\theta).
\end{aligned}
$$

3.2 Simulation Details

A series of y_i can be generated based on equation (2):

$$y_i = \mathbf{x}_i'\boldsymbol{\beta} + \epsilon_i, \quad \epsilon_i \sim D(0, \sigma^2).$$

Two predictor variables $\mathbf{x}_1$ and $\mathbf{x}_2$, i.e. k = 2, are simulated from a uniform (0,1) distribution, and the values of $\boldsymbol{\beta}$ are set to 10. The number of observations n = 500 or 1250. We choose distribution of $\boldsymbol{\epsilon}$ from two distributions: standard normal or t-distribution with 5 degrees of freedom.

Although normally distributed returns are assumed in the RiskMetrics model (Morgan and Reuters (1996)), a heavy tail is observed in many financial data (see Mina and Xiao (2001)). Different alternative distributions have been suggested, and t-distribution is a commonly used one; therefore, the true error is assumed to follow either one of these two distributions.

After selecting the true error distribution, we scale the variance of true error to control the coefficient of determination R^2:

$$\sigma^2 \simeq \frac{(1 - R^2)RSS}{nR^2},$$

where the regression sum of square $RSS = \sum_{i=1}^{n} (\hat{y}_i - \overline{y})^2$, which could be found by using the relation $(\hat{y}_i - \overline{y}) = (\mathbf{x}_\mathrm{i} - \bar{\mathbf{x}})'\boldsymbol{\beta}$. Finally, a series of y_i is then generated using equation (2).

We view R^2 as a measure of the signal strength in the model. If R^2 is too small, then (2) is reduced to a sole noise function; while if R^2 is too large, then only signals remain in (2). Both situations are not desirable for percentile estimation purposes, and so we set R^2 = 0.25 or 0.75. For each of the 8 models formed by different combinations of parameters and error distribution (n = 500 or 1250, $\epsilon \sim N$ or $\epsilon \sim t_5$ and R^2 = 0.25 or 0.75), **X** and **y** are generated for 1000 replications. Least square and quantile regression percentiles are then computed at probabilities θ = 0.01, 0.05, and 0.50 for 5 sets of **X** respectively. Here, $\mathbf{X} = [\mathbf{1}, \mathbf{0.5}, x_{2i}]$ where $\mathbf{1}, \mathbf{0.5}$ are vectors of 1s and 0.5s, respectively, and $x_{2i} = (0.1, 0.3, 0.5, 0.7, 0.9)$ for i = 1,...., 5.

We then calculate sample mean and standard deviation for least square and quantile regression percentiles. We also carry out the Shapiro Wilk W test to see if the estimation error is normally distributed.

3.3 Simulation Results

Tables 1-3 show the actual percentiles, average of least square percentiles, and average of quantile regression percentiles over 1000 replications as well as their standard deviations for different probabilities when n = 500 or 1250 for R^2 = 0.25 when the true error distribution is normal. Figures 1 and 2 show the percentiles graphically. For n = 500 and R^2 = 0.25, if the actual error term is normally distributed, then the least square method performs better than the quantile regression method, because the least square method has additional information on the normality of the true error term. The average of least square percentiles is closer to the actual percentile, and the least square sample standard deviation is smaller than that of quantile regression. Though not as good as least square one, the performance of quantile regression estimates improves when n increases. The normality tests for estimation errors in both methods are not rejected at the 5% significance level.

Table 1: Actual, LS, and QR Average Percentiles when θ = 0.01, $\epsilon \sim$ Normal, and R^2 = 0.25 with Standard Deviations of Percentiles Shown in Parenthesis

	n = 500			n = 1250		
i	Actual	LS	QR	Actual	LS	QR
1	-1.0568	-1.0360	-0.9238	-0.5868	-0.5946	-0.5793
		(0.5586)	(2.1835)		(0.3469)	(0.9197)
2	0.9432	0.9620	1.0811	1.4131	1.4100	1.4341
		(0.4049)	(1.5682)		(0.2483)	(0.9197)
3	2.9432	2.9600	3.0861	3.4132	3.4146	3.4475
		(0.3339)	(1.2247)		(0.2049)	(0.7535)
4	4.9432	4.9576	5.0910	5.4131	5.4192	5.4610
		(0.3933)	(1.3748)		(0.2475)	(0.9172)
5	6.9432	6.9559	7.0959	7.4132	7.4238	7.4744
		(0.5418)	(1.9051)		(0.3457)	(1.2909)

Tables 4-6 are similar to Tables 1-3 except that their true error distribution is *t*. Figures 3 and 4 present the percentiles graphically. When the true error term follows a *t*-distribution with 5 degrees of freedom, the average of $\hat{Q}_{QR,i}(\theta)$ is closer to the actual quantile than the average of $\hat{Q}_{LS,i}(\theta)$. The standard deviation of $\hat{Q}_{QR,i}(\theta)$ is larger than that of $\hat{Q}_{LS,i}(\theta)$ as expected, especially when θ is small, because the quantile regression method does not assume any distribution on the error term. When the sample size increases, the average of $\hat{Q}_{QR,i}(\theta)$ moves closer to the actual percentile (see the results in Table 6).

As expected, the normality tests for least square estimation errors are not rejected for all values of θ, while those for quantile regression are rejected at $\theta = 0.01$. For R^2 = 0.75, we obtain similar results, but the deviations between the statistics of estimated percentiles decrease.

Table 2: Actual, LS, and QR Average Percentiles when θ = 0.05, $\epsilon \sim$ Normal, and R^2 = 0.25 with Standard Deviations of Percentiles Shown in Parenthesis

		n = 500			*n* = 1250	
i	Actual	LS	QR	Actual	LS	QR
1	3.9399	3.9607	3.9781	4.2722	4.2645	4.2831
		(0.5586)	(1.1988)		(0.3469)	(0.7450)
2	5.9399	5.9587	5.9849	6.2722	6.2691	6.2852
		(0.4049)	(0.8577)		(0.2483)	(0.5260)
3	7.9399	7.9567	7.9918	8.2722	8.2736	8.2874
		(0.3339)	(0.6815)		(0.2049)	(0.4235)
4	9.9399	9.9547	9.9987	10.2722	10.2782	10.2895
		(0.3933)	(0.7897)		(0.2475)	(0.5130)
5	11.9399	11.9527	12.0055	12.2722	12.2828	12.2917
		(0.5418)	(1.1013)		(0.3457)	(0.7268)

Table 3: Actual, LS, and QR Average Percentiles when θ = 0.50, ϵ ~Normal, and R^2 = 0.25 with Standard Deviations of Percentiles Shown in Parenthesis

		n = 500			*n* = 1250	
i	Actual	LS	QR	Actual	LS	QR
1	16.0000	16.0208	16.0179	16.0000	15.9922	15.9925
		(0.5586)	(0.6870)		(0.3469)	(0.4295)
2	18.0000	18.0188	18.0198	18.0000	17.9968	18.0000
		(0.4049)	(0.5011)		(0.2483)	(0.3100)
3	20.0000	20.0168	20.0218	20.0000	20.0014	20.0072
		(0.3339)	(0.4268)		(0.2049)	(0.2584)
4	22.0000	22.0148	22.0238	22.0000	22.0060	22.0145
		(0.3933)	(0.5149)		(0.2475)	(0.3108)
5	24.0000	24.0128	24.0257	24.0000	24.0106	24.0219
		(0.5418)	(0.7072)		(0.3457)	(0.4307)

Table 4: Actual, LS, and QR Average Percentiles when θ = 0.01, $\epsilon \sim t$, and R^2 = 0.25 with Standard Deviations of Percentiles Shown in Parenthesis

		n = 500			*n* = 1250	
i	Actual	LS	QR	Actual	LS	QR
1	-3.1106	-1.0389	-3.3591	-2.5840	-0.5749	-2.7914
		(0.5581)	(3.9050)		(0.3405)	(2.4411)
2	-1.1106	0.9513	-1.3529	-0.5840	1.4197	-0.7736
		(0.3943)	(2.9439)		(0.2418)	(1.7741)
3	0.8894	2.9414	0.6532	1.4160	3.4142	1.2442
		(0.3237)	(2.5244)		(0.1978)	(1.4914)
4	2.8894	4.9316	2.6594	3.4160	5.4088	3.2620
		(0.3992)	(2.8927)		(0.2406)	(1.7860)
5	4.8894	6.9217	4.6656	5.4160	7.4033	5.2797
		(0.5649)	(3.8277)		(0.3388)	(2.4585)

Table 5: Actual, LS, and QR Average Percentiles when $\theta = 0.05$, $\epsilon \sim t$, and $R^2 = 0.25$ with Standard Deviations of Percentiles Shown in Parenthesis

		n = 500			n = 1250	
i	Actual	LS	QR	Actual	LS	QR
1	4.5558	3.9579	4.4684	4.8712	4.2841	4.8413
		(0.5581)	(1.4981)		(0.3405)	(0.9252)
2	6.5558	5.9480	6.4806	6.8712	6.2787	6.8411
		(0.3943)	(1.0743)		(0.2418)	(0.6489)
3	8.5558	7.9382	8.4928	8.8712	8.2733	8.8408
		(0.3237)	(0.8643)		(0.1978)	(0.5274)
4	10.5558	9.9283	10.5050	10.8712	10.2678	10.8406
		(0.3992)	(1.0114)		(0.2406)	(0.6537)
5	12.5558	11.9184	12.5172	12.8712	12.2624	12.8404
		(0.5649)	(1.4077)		(0.3388)	(0.9321)

Table 6: Actual, LS, and QR Average Percentiles when $\theta = 0.50$, $\epsilon \sim t$, and $R^2 = 0.25$ with Standard Deviations of Percentiles Shown in Parenthesis

		n = 500			n = 1250	
i	Actual	LS	QR	Actual	LS	QR
1	16.0000	16.0180	16.0381	16.0000	16.0119	16.0103
		(0.5581)	(0.5566)		(0.3405)	(0.3627)
2	18.0000	18.0081	18.0285	18.0000	18.0065	18.0092
		(0.3943)	(0.4051)		(0.2418)	(0.2571)
3	20.0000	19.9983	20.0188	20.0000	20.0010	20.0081
		(0.3237)	(0.3416)		(0.1978)	(0.2093)
4	22.0000	21.9884	22.0092	22.0000	21.9956	22.0070
		(0.3992)	(0.4093)		(0.2406)	(0.2542)
5	24.0000	23.9785	23.9996	24.0000	23.9901	24.0059
		(0.5649)	(0.5627)		(0.3388)	(0.3586)

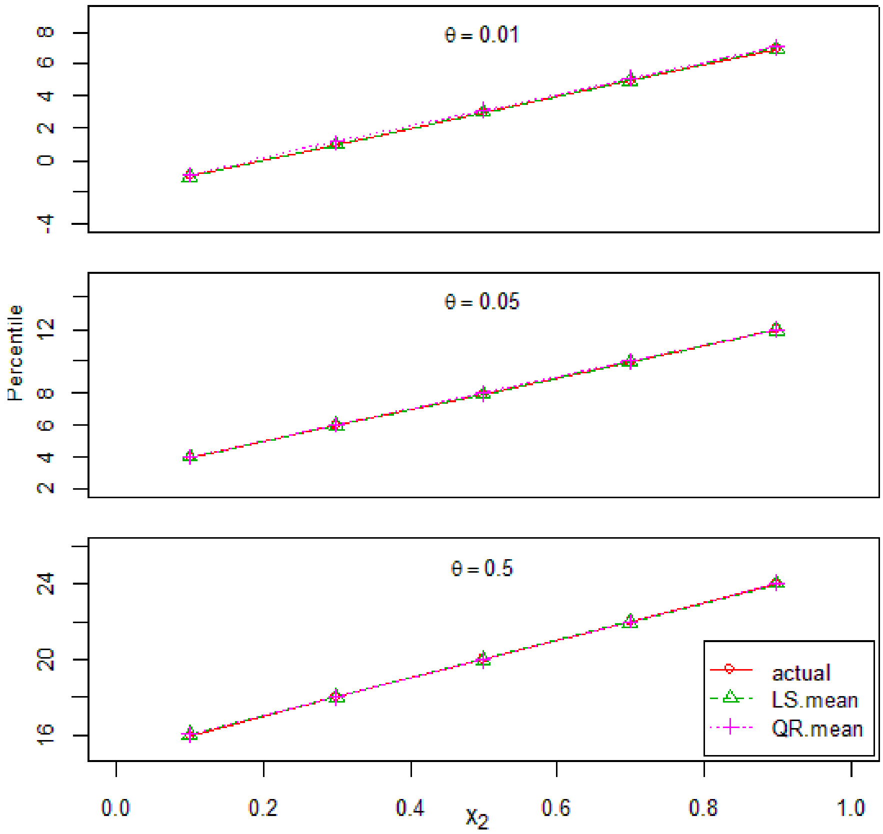

Figure 1: Plots of Actual, LS, & QR Average Percentiles against x_2 when R^2 = 0.25, n = 500, and ϵ ~ Normal

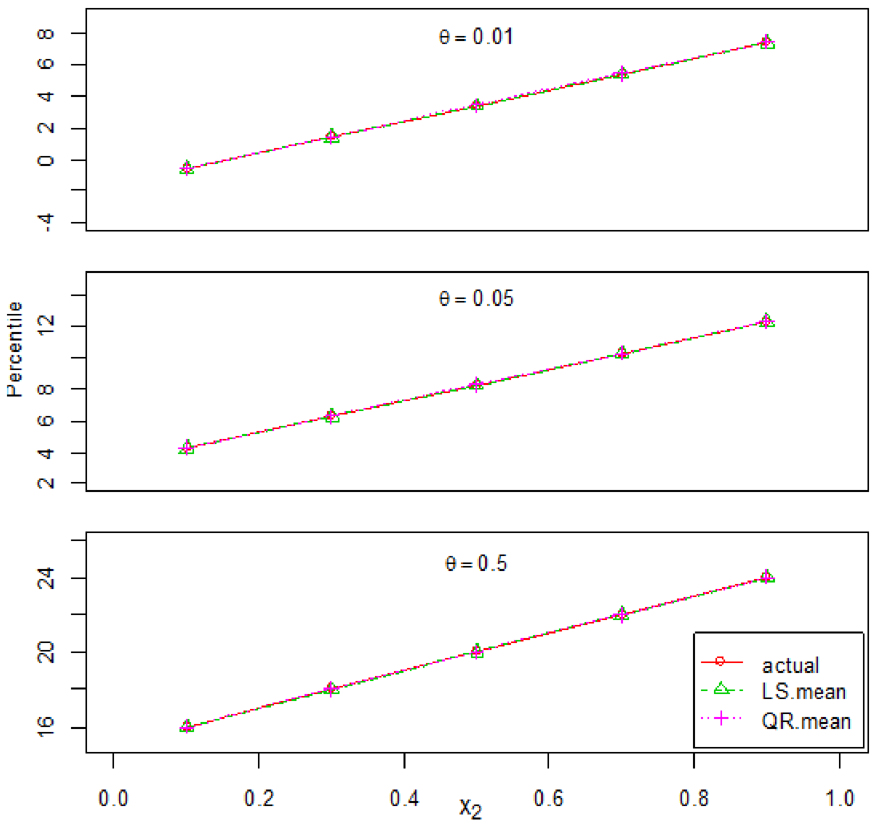

Figure 2: Plots of Actual, LS, & QR Average Percentiles against x_2 when R^2 = 0.25, n = 1250, and ϵ ~ Normal

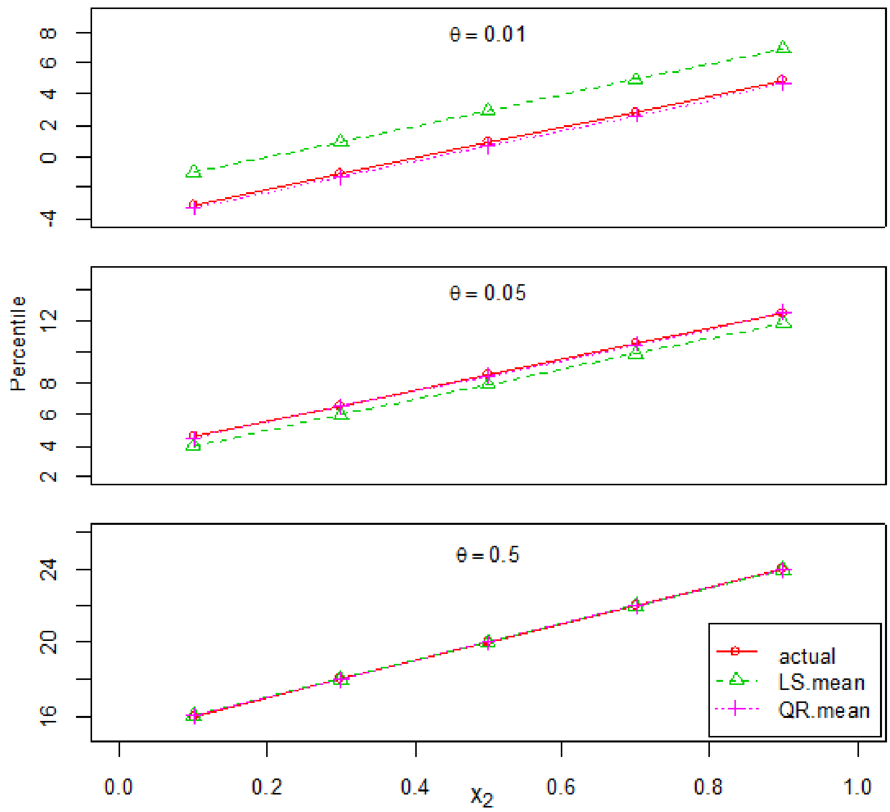

Figure 3: Plots of Actual, LS, & QR Average Percentiles against x_2 when R^2 = 0.25, n = 500, and $\epsilon \sim t$

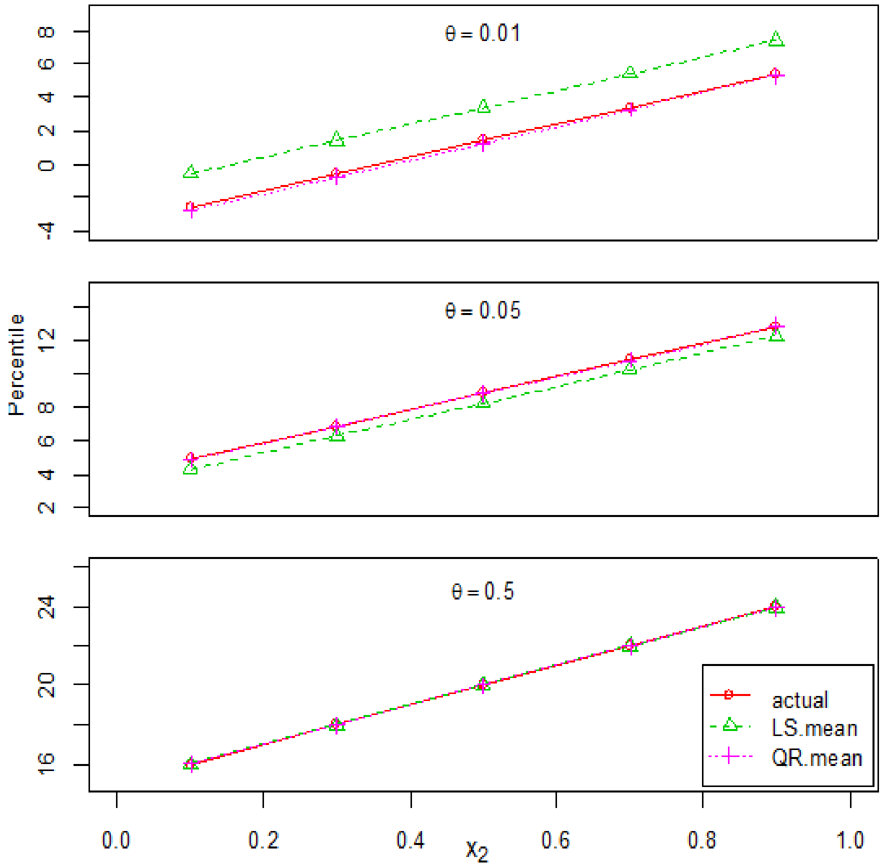

Figure 4: Plots of Actual, LS, & QR Average Percentiles against x_2 when R^2 = 0.25, n = 1250, and $\epsilon \sim t$

4 VaR Estimation Approaches

This section discusses the several Value at Risk (VaR) estimation methods being used. Our study considers three estimation approaches: 1) RiskMetrics, 2) empirical distribution, and 3) quantile regression.

Let p_t be the price index at time t and Ω_t be the information up to time t. The continuous compounded daily return at time t is $r_t = \log(p_t / p_{t-1})$. The aggregate return at time t for a time horizon h is $R_{t,h} = \log(p_{t+h} / p_t)$.

VaR measures the loss of a portfolio over the time horizon h with probability θ. It can be expressed in terms of the portfolio return and formulated as:

$$P(R_{t,h} < VaR_{t,h}(\theta)) = \theta,$$

where $VaR_{t,h}(\theta)$ is the VaR value of aggregate returns $R_{t,h}$. Recall that the Basel Committee has set h = 10 and θ = 0.01.

According to Taylor (1999), volatility estimation and the assumption of probability distribution are usually involved in the multiple period quantile estimation procedure. Given Ω_t, we denote the conditional variance of r_{t+1} by $var(r_{t+1}|\Omega_t) = \sigma_{t+1}^2$ and the conditional variance of $R_{t,h}$ by $var(R_{t,h}|\Omega_t)$. Two methods are commonly used in a multiple period volatility forecast. The first one is the exponentially weighted moving average (EWMA) model used in Riskmetrics (Morgan and Reuters (1996)):

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)r_t^2, \tag{3}$$

and $var(R_{t,h}|\Omega_t) = h\sigma_{t+1}^2$.

When the decay factor λ is smaller, the weight given to recent events is greater.

The other method is the generalized autoregressive conditional heteroscedastic (GARCH) model proposed by Bollerslev (1986). Fixing the conditional mean at 0, we have the following GARCH(p,q) process:

$$\begin{aligned} r_t &= \sigma_t\epsilon_t, \qquad\qquad \epsilon_t|\Omega_{t-1} \sim D(0,1), \\ \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{q}\alpha_i r_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 , \end{aligned}$$

where $D(0,1)$ represents a distribution with mean 0 and variance 1. In this study, $D(0,1)$ is either a normal distribution or a t-distribution with degrees of freedom *df*. For the GARCH(1,1) process, the conditional variance of r_{t+1} is:

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 r_t^2 + \beta_1\sigma_t^2. \tag{4}$$

Letting $\phi_1 = \alpha_1 + \beta_1$, the aggregate conditional variance is (see Wong and So (2003)):

$$var(R_{t,h}|\Omega_t) = \begin{cases} \dfrac{\alpha_0}{1-\phi_1}[h - \dfrac{1-\phi_1^h}{1-\phi_1}] + \dfrac{1-\phi_1^h}{1-\phi_1}\sigma_{t+1}^2 & \text{if } \phi_1 < 1 \\ \dfrac{(h-1)h}{2}\alpha_0 + h\sigma_{t+1}^2 & \text{if } \phi_1 = 1 \end{cases}.$$

Note that the RiskMetrics approach is a special case of the GARCH model (IGARCH), where $\alpha_0 = 0$, $\alpha_1 = 1 - \lambda$, $\beta_1 = \lambda$, and $\phi_1 = 1$, and $D(0,1)$ is a standard normal distribution. All three estimation approaches discussed below use the volatility forecasts.

4.1 RiskMetrics Approach

In the RiskMetrics estimation approach, the conditional mean of r_t is suggested to be zero and the conditional distribution of ϵ_t is assumed to be normal:

$$r_t = \sigma_t \epsilon_{t,} \quad \epsilon_t|\Omega_{t-1} \sim N(0,1).$$

The θ^{th} quantile of $R_{t,h}$ is therefore:

$$\begin{aligned} VaR_{t,h}(\theta) &= \Phi^{-1}(\theta) var(R_{t,h}|\Omega_t) \\ &= \Phi^{-1}(\theta)\sqrt{h}\sigma_{t+1}, \end{aligned}$$

where $\Phi(\theta)$ is the cumulative distribution function of a standard normal random variable. As suggested by Morgan and Reuters (1996), the decay factor λ in equation (3) is set at 0.94 for daily data and 0.97 for monthly data.

4.2 Empirical Distribution Approach

This approach is based on the empirical distribution of standardized returns (see Taylor (1999)). We assume the conditional returns can be estimated by the empirical returns in the sample window. The standardized return is the division of the aggregate return by the volatility forecast - for example, the standardized h-period return is:

$$sr_{t,h} = \frac{R_{t,h}}{\sqrt{var(R_{t,h}|\Omega_t)}}.$$

The sample quantile $\hat{q}_{sr}(\theta)$ can be obtained from the empirical distribution of standardized returns, and so the θ^{th} quantile of $R_{t,h}$ is:

$$VaR_{t,h}(\theta) = \hat{q}_{sr}(\theta)\sqrt{var(R_{t,h}|\Omega_t)}.$$

4.3 Quantile Regression Approach

One important issue involved in regression analysis is the selection of response variable y and covariates $x_{1,}\dots,x_k$. These covariates should have the power to predict stock returns in our study. Past returns and fundamental variables are two general types of "predictable" component in stock returns (see Chernozhukov and Umantsev (2001), Kaul (1996), Patelis (1997)). Lag returns r_{t-1} and volatility forecast σ_{t+1} are examples of past returns variables, while dividend yield and interest rate are examples of fundamental variables. The θ^{th} quantile of $R_{t,h}$ is:

$$VaR_{t,h}(\theta) = \mathbf{x}_t'\hat{\boldsymbol{\beta}}(\theta),$$

where $\hat{\boldsymbol{\beta}}(\theta)$ is the vector of quantile regression coefficients obtained from equation (1).

4.4 Evaluation Methods

Different methods have been suggested for comparing the VaR estimation approaches - for example, Christoffersen, Hahn and Inoue (2001). This paper considers two methods. Let's define:

$$I_t = \begin{cases} 1, & \text{if } R_{t,h} < VaR_{t,h}(\theta) \\ 0, & \text{if } R_{t,h} \geq VaR_{t,h}(\theta) \end{cases}$$

and N is the total number of $VaR_{t,h}(\theta)$ in the evaluation process.

One commonly used method is calculating the ratio $\hat{p}/p$ (Basle (1996b)), $\hat{p} = \sum_{t=1}^{N} I_t/N$, where $\hat{p}$ is the proportion of $R_{t,h}$ falling below $VaR_{t,h}(\theta)$, and p is the true proportion - that is, $p = \theta$. This test assumes $\{I_t\}$ are independent. If the VaR estimation approach is good, then we expect this ratio should be close to one.

Moreover, under the Basel Accord, estimation approaches that overestimate risk $\left(\hat{p}/p < 1\right)$ are preferable to those that underestimate risk levels.

An extension of the above test is the likelihood ratio (LR) tests proposed by Christoffersen (1998). Three LR tests are suggested: LR test of correct unconditional coverage, LR test of independence, and LR test of correct conditional coverage. We expect these hypothesis tests will not be rejected for a good VaR estimate.

In the LR test of correct unconditional coverage, we let:

$$n_0 = \sum_t (I_t = 0), \quad n_1 = \sum_t (I_t = 1) \qquad t = 2, \ldots, N,$$

and $\hat{\pi} = \frac{n_1}{N-1}$. Assume $\{I_t\}$ are independent. The hypotheses are:

H_0 : $\mathbf{E}[I_t] = p$
H_1 : $\mathbf{E}[I_t] \neq p$

Here, the LR is:

$$LR_{uc} = -2[n_0 \log(\frac{1-p}{1-\hat{\pi}}) + n_1 \log(\frac{p}{\hat{\pi}})] \overset{H_0}{\sim} \chi^2(1).$$

The LR test of unconditional coverage is similar to the ratio $\hat{p}/p$ test. The unconditional coverage rate of a good estimate will be close to probability θ. However, this test is unable to capture the information that $I_t = 0$ or $I_t = 1$ may cluster together in a time-dependent structure, and so the LR test of independence is suggested.

In the LR test of independence, we let:

$$n_{00} = \sum_t (I_t = 0 | I_{t-1} = 0), \qquad n_{01} = \sum_t (I_t = 0 | I_{t-1} = 1)$$
$$n_{10} = \sum_t (I_t = 1 | I_{t-1} = 0), \qquad n_{11} = \sum_t (I_t = 1 | I_{t-1} = 1), \ t = 2, \ldots, N,$$

and we define $\hat{\pi}_{01} = \frac{n_{01}}{n_{00}+n_{01}}$, $\hat{\pi}_{11} = \frac{n_{11}}{n_{10}+n_{11}}$. The hypotheses are:

H_0 : $\{I_t\}$ are independent
H_1 : $\{I_t\}$ are dependent

Here, the LR is:

$$\begin{aligned} LR_{ind} \quad &= \quad -2[n_{00}\log(\frac{1-\hat{\pi}}{1-\hat{\pi}_{01}}) + n_{10}\log(\frac{1-\hat{\pi}}{1-\hat{\pi}_{11}}) \\ &\quad + n_{01}\log\frac{\hat{\pi}}{\hat{\pi}_{01}} + n_{11}\log\frac{\hat{\pi}}{\hat{\pi}_{11}}] \overset{H_0}{\sim} \chi^2(1). \end{aligned}$$

The LR test of conditional coverage is the combination of the above two LR tests and will be used to evaluate the VaR estimates in the empirical study. Its hypotheses are:

H_0 : $\mathbf{E}[I_t] = p$ and $\{I_t\}$ are independent
H_1 : $\mathbf{E}[I_t] \neq p$ or $\{I_t\}$ are dependent

Here, the LR is:

$$\begin{aligned} LR_{cc} \quad &= \quad -2\log\frac{(1-p)^{n_0}p^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}}(1-\hat{\pi}_{11})^{n_{10}}(\hat{\pi}_{01})^{n_{01}}(\hat{\pi}_{11})^{n_{11}}} \\ &= \quad LR_{uc} + LR_{ind} \overset{H_0}{\sim} \chi^2(2). \end{aligned}$$

5 Empirical Study

We compare the VaR estimates calculated under different approaches by using data on four stock market indices. Daily data for the four international stock market indices of Hong Kong, Japan, the UK, and the U.S. are obtained from Datastream. Four years of data are collected from January 2, 1997 (January 6, 1997 for Japan) until December 29, 2000 (December 30, 2000 for the UK). There are roughly 1000 observations in each series. To examine the performance of the approaches under highly varied market conditions, we select this period as it covers the Asian financial crisis (AFC). AFC had a serious impact on Asian stock markets (e.g. HSI dropped 23% between 20 and 23 October, 1997), but had a relatively mild effect on Europe and U.S. markets. Daily returns are calculated, and their time series plots are shown in Figure 5. Table 7 provides the summary statistics, mean, standard deviation (Sd), skewness (Skew), and excess kurtosis (EKurt) of the daily returns.

Table 7: Summary Statistics of Stock Indices' Daily Returns for the Period 1997-2000

Index (Country or City)	Size	Mean $\times 10^{-4}$	Sd	Skew	EKurt
FTSE 100 (UK)	1008	4.2423	0.0116	-0.1181	0.6174
HSI (Hong Kong)	985	1.3596	0.0229	0.2278	7.3086
NIKKEI 225 (Japan)	984	-3.4960	0.0156	0.0151	1.9144
S&P 500 (U.S.)	1008	5.7837	0.0125	-0.3097	3.0766

Among all stock market indices, the Japan market is the only one that had a negative average return over the sample period, while the S&P 500 had the highest average daily return. Daily returns of the Hong Kong market were most volatile, while those of the FTSE 100 were least volatile. The S&P 500 had a highly skewed return distribution, while the NIKKEI 225 had quite a symmetric distribution. The daily returns of all stock markets had a heavier tail than a normal distribution, coinciding with what we stated in the Introduction. The distribution of HSI returns had the largest excess kurtosis among all, followed by that of the U.S. market. The large excess kurtosis of HSI may be due to the outliers around the Asian financial crisis in 1997. If we delete the data for the period July 1997 to March 1998, then HSI's excess kurtosis drops significantly from 7.3086 to 4.9849.

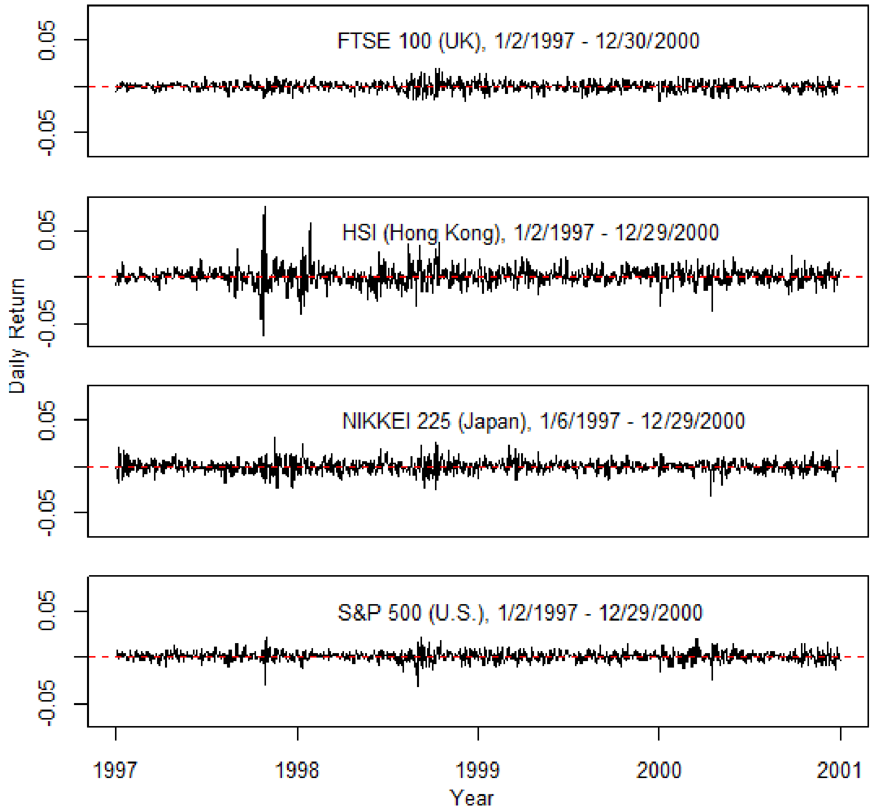

Figure 5: Time Series Plots of the Daily Returns of Stock Indices FTSE 100, HSI, NIKKEI 225, and S&P 500 for the Period 1997-2000

5.1 Estimation Procedures

This subsection shows the procedures for calculating and comparing the VaRs. We consider three models to estimate VaR:

- **riskme**: RiskMetrics with normally distributed ϵ_t.
- **empes**: Empirical distribution using EWMA (RiskMetrics) volatility forecast.
- **qreg-t**: Quantile regression using GARCH(1,1) volatility forecast model with *t*-distributed ϵ_t.

We estimate the VaR for holding period *h* of length one, three, five, seven, ten, twelve, and fifteen days at probabilities θ = 0.01, 0.025, and 0.05, respectively, at time *t*. We then calculate the actual aggregate returns for *h* = 1, 3, 5, 7, 10, 12 and 15 from the daily returns of market indices and compare them with the VaR estimates. We perform a rolling window analysis of these VaR estimation procedures.

In this study we set the rolling window size of 500 observations. The in-sample data serves four purposes: (1) estimate the GARCH parameters in the GARCH volatility forecast model; (2) set the initial conditional variance for all estimation approaches; (3) construct the empirical distribution of aggregate returns in the empirical distribution approach; and (4) select the predictor variables in the quantile regression.

The decay factor λ used in the EWMA volatility forecast equation is set as 0.94 as suggested by RiskMetrics for daily data, while the GARCH parameters (α_0, α_1, β_1, *df*) in equation (4) are estimated using the maximum likelihood approach, where *df* is the degrees of freedom of the t-distribution. We begin our first estimation at *t* = 500. To initiate the volatility forecast method, we use the sample variance of the daily returns in the estimation window as the initial conditional variance for EWMA and the unconditional variance of r_{t+1} of GARCH(1,1):

$$\sigma^2 = \frac{\alpha_0}{1 - \alpha_1 - \beta_1},$$

as the initial conditional variance for GARCH.

We next compute the VaR estimate and the actual multiple period return at the end time point. We then drop one return data from the beginning of the sample and add a new return to the end of the sample, which is equivalent to moving the sample window to the right. In the new window, the initial conditional variance and the empirical distribution of aggregate returns are renewed. Though the GARCH parameters are supposed to be re-estimated everyday, to save computational time we re-estimate every 5 moving windows. We find that the volatility estimates calculated using the GARCH parameters re-estimated daily, every 5 days, or every 20 days do not differ a lot, but there is a significant reduction in computational time for the 5-day re-estimation compared with the daily re-estimation. Table 8 presents some volatility estimates for different re-estimation periods using the S&P 500 data.

Table 8: Conditional Volatility Estimates for Different Re-estimation Periods

	$\sigma_{t+1(\times 10^{-2})}$		
t	daily	5 days	20 days
500	1.2142	1.2142	1.2142
504	1.1180	1.1171	1.1171
509	1.1432	1.1388	1.1383
514	1.3613	1.3478	1.3555
519	1.2264	1.2280	1.2175

The whole procedure is repeated until the whole validation period (1999-2000) is covered. We adopt the evaluation methods mentioned in Section 4.4 to compare the estimation methods.

A modification is required for the LR tests when they are used in the empirical study. The LR test of independence and the LR test of conditional coverage are very likely to fail in most cases except for the single holding period, because the LR tests have not been modified to suit the multiple period VaR case. The aggregate returns are dependent with subsequent ones, because they are formed by overlapping the daily returns data r_t. For example, if h = 10, $R_{500,10}(\theta)$ and $R_{501,10}(\theta)$ are overlapping aggregate returns, whereas $R_{500,10}(\theta)$ and $R_{510,10}(\theta)$ are not. Even though the conclusion may deviate from the actual one, to give some sense of the performance, we instead compare the "non-overlapping" $R_{t,h}$ with the corresponding $VaR_{t,h}(\theta)$. The number of "non-overlapping" aggregate returns, n_{non}, decreases when the holding period increases. Table 9 shows the number of "non-overlapping" $R_{t,h}(\theta)$ from a series of 1008 daily returns for different holding periods.

Table 9: Number of "Non-overlapping" Aggregate Returns for Different Holding Periods for the Daily Returns Data

h	1	3	5	7	10	12	15
n_{non}	494	165	99	71	50	42	33

5.2 Predictors in the Quantile Regression Approach

Section 4.3 has suggested some possible predictors for stock index returns. For the sake of simplicity, we shall use past return type variables in this study. As suggested by Taylor (1999), aggregate returns can be the response variable and different combinations of the power of holding period and conditional volatility of r_{t+1} can be considered as predictive variables. To illustrate this idea, suppose the predictors are holding period h and conditional volatility σ_{t+1}. The aggregate returns series and predictors series are then:

$$
\begin{array}{lccc}
 & R_{t,h} & h & \sigma_{t+1} \\
\text{1-period returns} & \left\{\begin{array}{c} R_{1,1} \\ R_{2,1} \\ \vdots \\ R_{1249,1} \end{array}\right. & \begin{array}{c} 1 \\ 1 \\ \vdots \\ 1 \end{array} & \begin{array}{c} \sigma_2 \\ \sigma_3 \\ \vdots \\ \sigma_{1250} \end{array} \\
\text{3-period returns} & \left\{\begin{array}{c} R_{1,3} \\ R_{2,3} \\ \vdots \\ R_{1247,3} \end{array}\right. & \begin{array}{c} 3 \\ 3 \\ \vdots \\ 3 \end{array} & \begin{array}{c} \sigma_2 \\ \sigma_3 \\ \vdots \\ \sigma_{1248} \end{array} \\
\vdots & & \vdots & \vdots \\
\text{15-period returns} & \left\{\begin{array}{c} R_{1,15} \\ R_{2,15} \\ \vdots \\ R_{1235,15} \end{array}\right. & \begin{array}{c} 15 \\ 15 \\ \vdots \\ 15 \end{array} & \begin{array}{c} \sigma_2 \\ \sigma_3 \\ \vdots \\ \sigma_{1236} \end{array}
\end{array}
$$

We consider a wide range of predictors: $\sigma_{t+1}, \sqrt{h}, \sigma_{t+1}^2, h\sigma_{t+1}, \sqrt{h}\sigma_{t+1}, h\sigma_{t+1}^2, h^2\sigma_{t+1}$ and $h^2\sigma_{t+1}^2$. The quantile regression goodness of fit (Koenker and Machado (1999)) is used to select the predictors.

The quantile regression goodness of fit, also named as $pseudo - R^2$, is similar to the coefficient of determination R^2 in the least square regression:

$$pseudo - R^2 = 1 - \frac{v_1}{v_0},$$

where ν_0 is the quantile function value about the empirical quantile of y, and ν_1 is the quantile function value about the estimated regression quantile. Here, ν_0 can be viewed as the quantile function value about the estimated regression quantile without any information of predictive variables. Equation (1) can be used to calculate ν_1, while ν_0 can be calculated using the following equation:

$$\nu_0 = \min_{\beta}\left[\sum_{y\in\{i:y_i\geq\beta\}}\theta|y_i-\beta| + \sum_{y\in\{i:y_i<\beta\}}(1-\theta)|y_i-\beta|\right].$$

The value of $pseudo - R^2$ ranges from 0 to 1. We select the variable set with the highest $pseudo - R^2$.

We utilize the 500 observations in the first estimation window of the S&P 500 data (from January 2, 1997 to December 24, 1998) for variable selection and use the best subsets selection method with the selection criteria of highest $pseudo - R^2$. The conditional volatilities of daily returns σ_{t+1} are calculated using the GARCH volatility forecast model with ϵ_t following a *t*-distribution. Tables 10 and 11 show the variable sets with highest $pseudo - R^2$ among all in the groups with the same number of predictors. Table 11 indicates the one with 3 predictors $\sqrt{h}, h\sigma_{t+1}$, and $h^2\sigma_{t+1}^2$ has the highest $pseudo - R^2$.

Table 10: Estimated Parameters and $pseudo - R^2$ for Quantile Regression Models with Different Numbers of Predictors

Variable	k			
	8	7	6	5
constant	0.03	-0.00	-0.05	-0.01
σ_{t+1}	-5.97	-3.64	3.30	-1.09
$\sqrt{h}$	-0.04	-0.00	0.02	-0.00
σ_{t+1}^2	144.79	64.36	-135.82	
$h\sigma_{t+1}$	-0.89	-1.40	-1.54	-1.01
$\sqrt{h}\sigma_{t+1}$	3.89	2.44		
$h\sigma_{t+1}^2$	54.96	-35.59	1.32	-2.50
$h^2\sigma_{t+1}$	0.02			
$h^2\sigma_{t+1}^2$	3.29	4.84	4.08	3.11
$pseudo - R^2$	8.7	9.2	9.0	9.3

Table 11: Estimated Parameters and $pseudo-R^2$ for Quantile Regression Models with Different Numbers of Predictors

Variable	k			
	5	4	3	2
constant	-0.02	-0.02	-0.03	-0.02
σ_{t+1}				
$\sqrt{h}$	0.01	0.00	0.01	
σ_{t+1}^2	-17.66			
$h\sigma_{t+1}$	-1.21	-1.13	-1.34	-1.02
$\sqrt{h}\sigma_{t+1}$				
$h\sigma_{t+1}^2$	-0.89	-3.29		
$h^2\sigma_{t+1}$				
$h^2\sigma_{t+1}^2$	3.35	3.33	3.57	2.90
$pseudo-R^2$	9.3	9.4	9.6	9.3

Based on the evaluation results, we use $\sqrt{h}, h\sigma_{t+1}$ and $h^2\sigma_{t+1}^2$ as the predictors of the quantile regression approach for the empirical study. To simplify the variable selection process, we apply same variables selected for the S&P data to other stock indices in our study.

5.3 Results

The LR test of correct conditional coverage is not rejected at the 5% significance level for all the estimation methods and stock indices. In Figures 6-9, the ratios $\hat{p}/p$ applying different VaR estimation methods are plotted against holding period h for various stock indices. We prefer to choose the VaR estimation method with the ratio $\hat{p}/p$ closest to 1 and less than 1. Overall speaking, the performance of the estimation methods varies across stock indices, and no method surpasses others in all cases. We find that the ratios $\hat{p}/p$ usually move closer to 1 when θ gets larger. This is not surprising, because estimating the quantile near the tail distribution is more difficult.

When θ = 0.5, all three methods have similar results. We focus on the results of θ = 0.01 as this is the θ value specified by the Basel Accord. The quantile regression approach with GARCH-*t* conditional volatility performs well in FTSE 100 and S&P

500, because the ratios $\hat{p}/p$ are less than and close to 1 most of the time while the other two methods have ratios well above 1. In the HSI, all the three methods perform badly, possibly due to the presence of outliers that inflated the conditional volatility estimates. In the NIKKEI 225, all the three methods again perform badly. There is a trend that $\hat{p}/p$ increases with h when θ = 0.01 for all methods. This means that all VaR estimators are too confident and underestimate the VaR. The market risk capital charge may not be adequate if any of these estimation approaches is used.

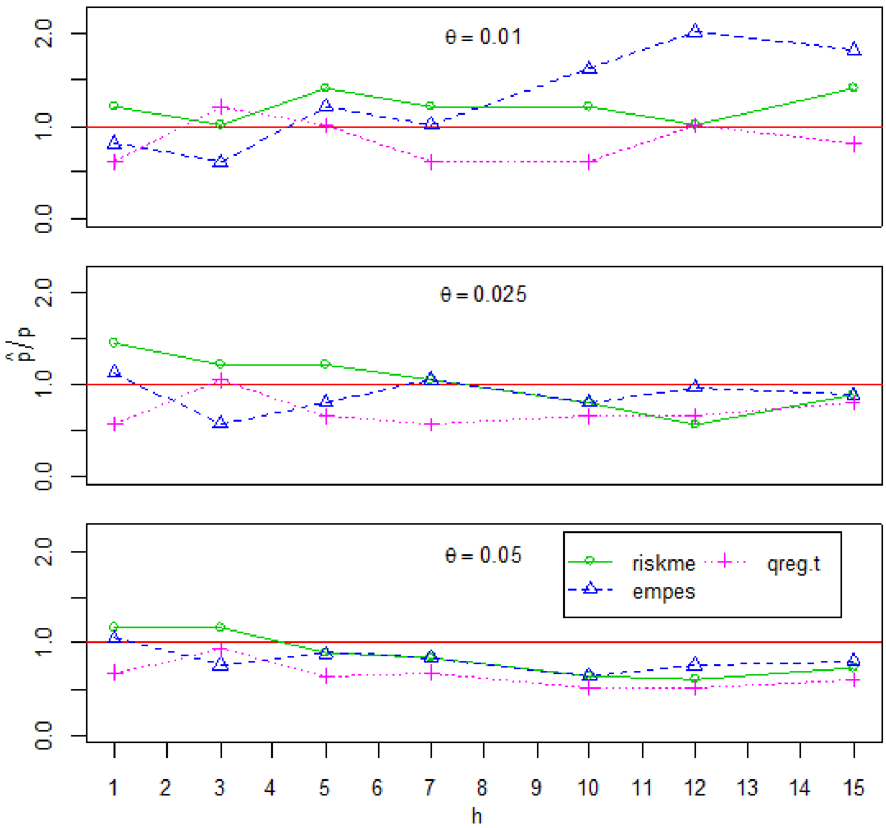

Figure 6: FTSE 100 - Plots of $\hat{p}/p$ against *h* for Three VaR Estimation Models

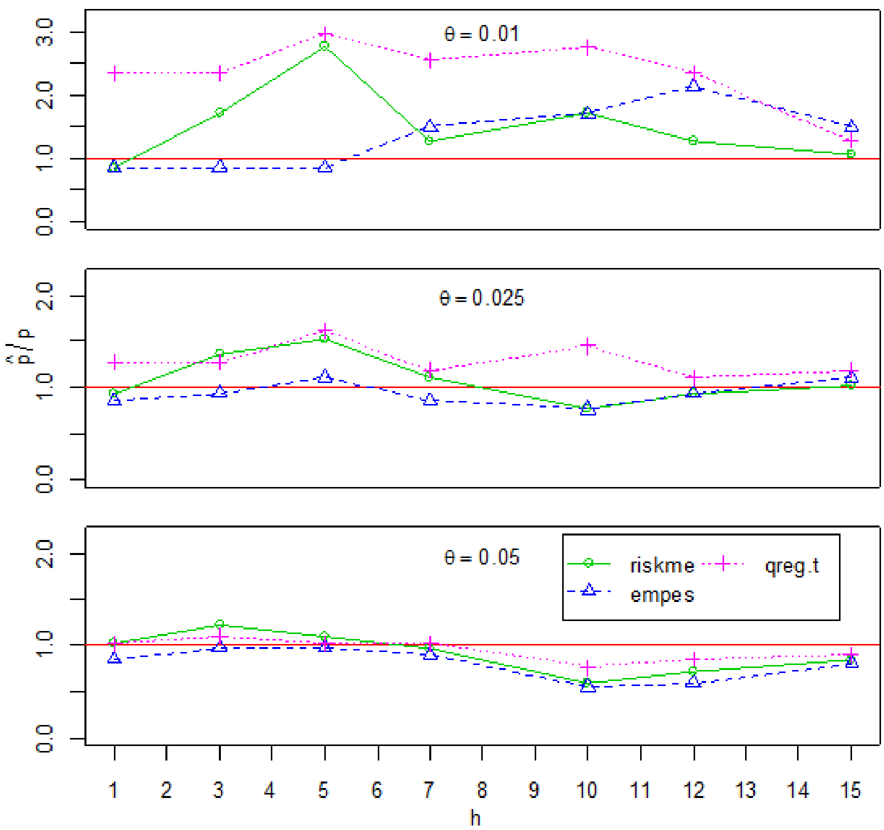

Figure 7: HSI - Plots of $\hat{p}/p$ against h for Three VaR Estimation Models

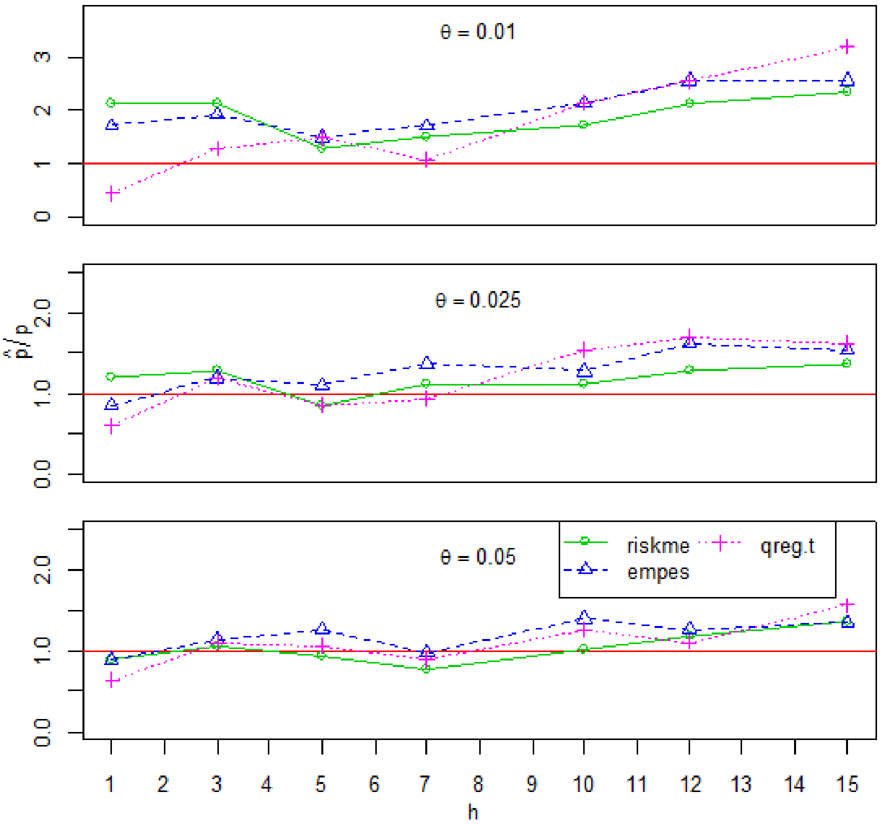

Figure 8: NIKKEI 225 - Plots of $\hat{p}/p$ against *h* for Three VaR Estimation Models

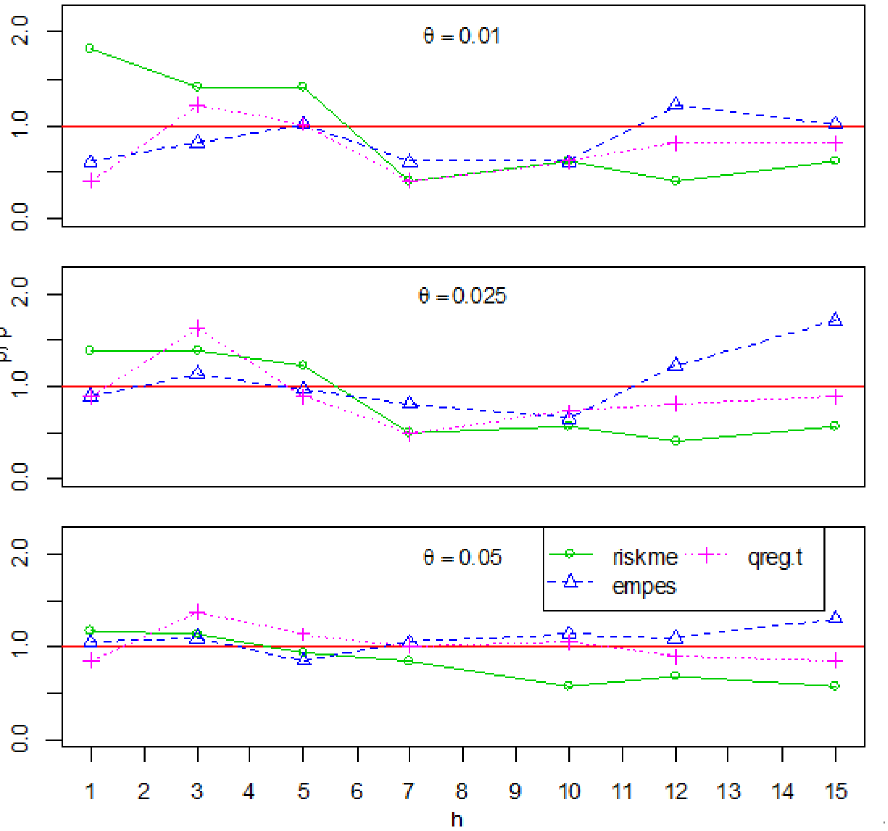

Figure 9: S&P 500 - Plots of $\hat{p}/p$ against h for Three VaR Estimation Models

6 Conclusion

This empirical study has applied VaR estimation approaches to the aggregate returns of some major market indices in Asia, Europe, and North America. The results indicate that the quantile regression approach is better than the other two approaches in the sense that it is less likely to underestimate the VaR. Our study also demonstrates that some appropriate predictive variables may be helpful in quantile regression-type VaR estimation. In the future, it would be worth investigating other variables, such as macroeconomic variables and conditional kurtosis, which can capture the market information and are good predictors of stock returns. The results of this article provide very useful guidance to financial risk analysts. In addition, the practical implementation of the proposed approach is a relatively straightforward exercise.

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